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MOST HYDRAULIC EFFICIENCY FOR OPEN CHANNEL SECTIONS OF HORIZONTAL BED WIDTH AND TWO SIDES

Abubaker M. Abusbeaa

Civil Engineering Department, Faculty of Engineering,
University of Tripoli, Libya.
E-mail: aabusbeaa@yahoo.com

الملخص

هذه الدراسة أظهرت معياراً جديداً للشروط المطلوبة لحساب المقطع الأكفأ هيدروليكياً لقنوات لها قاعدة أفقية وجانبين بميول مختلفة لكل منها. هذا المحدد الجديد محصور فقط للقنوات التي لا تتعرض للنحر ولا يتغير شكل مقطعها.

ABSTRACT

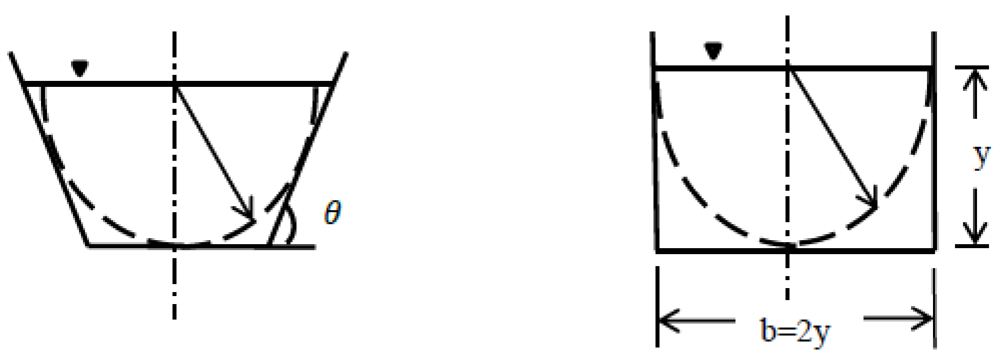
The study shows a new criterion for conditions required in determining most efficient flow section of a shape bounded by a horizontal base width and two sides of any slope for each of them. This new concept is limited for non-erodible prismatic channel.

KEYWORDS: Open Channel; Hydraulic Efficient Section; Best Flow Section; Channel Cross-Section Optimization

INTRODUCTION

Most efficient section; sometimes referred as best section or economy section; is a section optimized to give a maximum rate of discharge, $Q_{max.}$, for a flow area, A, or a section passes a given discharge with a minimum area of flow provided that longitudinal bed slope, S_0., and material roughness are kept constants. It should be clear that the efficiency meant is a hydraulic efficiency. An efficient section has a least resistance, i.e., flow witted perimeter P is minimum or hydraulic radius R_h=(A/P) is a maximum.
Among all cross-sections semi-circular section has the least witted perimeter for the same flow area and not necessarily the most economy section in practice.

It can be shown that the best trapezoidal shape is that which approximates most closely to a semi-circle, having its center in the surface, can be inscribed in the trapezoid, [1-3], in this case the best section conditions are: see Figure (1).



(a) Trapezoidal **(b) Rectangular**

Figure 1: Channel Sections of "Optimum" Shape [1]

i. $R_h = \frac{y}{2}$ (1)

ii. $Top\ width\ T = twice\ side\ slope\ lenght$

$$b + 2my = 2y\sqrt{m^2 + 1} \tag{2}$$

iii. $$\theta = \frac{\pi}{3} \tag{3}$$

Where: b= channel bed width

y = flow depth $\qquad m = cot\theta = {}^{1}/_{\sqrt{3}}$

θ = angle side of the trapezoid makes with horizontal

The Trapezoidal becomes half hexagon with each side = b

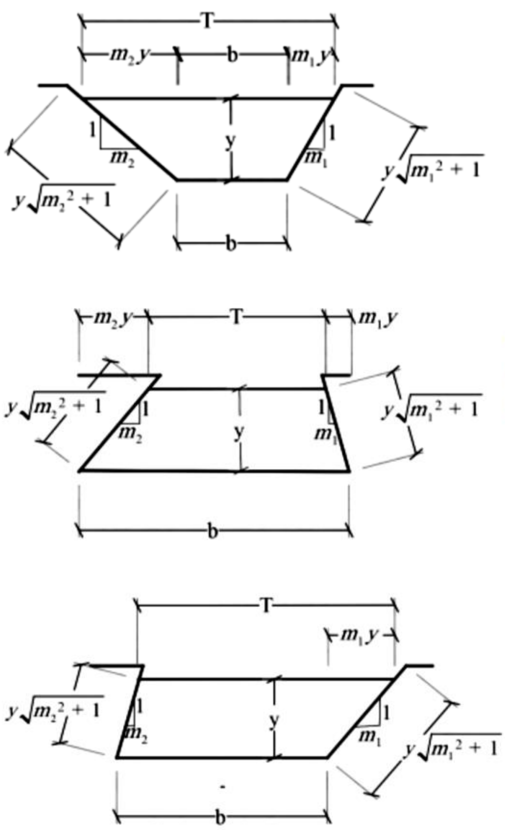
The semi-circle inscribed in the trapezoid contains about 91% of the trapezoid area which means that the difference between the two areas is about 9% only, and it is the least amount that can be reached. As m differs from $1/\sqrt{3}$, the difference in the two areas will be much more.

In the special case when the trapezoid is a rectangle, the best section is that for which the top width is twice the flow depth and the hydraulic radius is half the flow depth, see Figure (1b).

THEORY AND ANALYSIS

A trapezoidal cross-section is a symmetric shape with respect to its vertical axis; both sides have the same angle of inclination θ. How about if two sides have different slope such as m_1 *and* m_2? $m_1 = \cot\theta_1$ and $m_2 = \cot\theta_2$, $\theta_1 \neq \theta_2$

Five possible cases of flow sections can be detected as shown in Figure (2); distorted trapezoidal sections.



Case (a)
both sides outwarded with respect to (w.r.t) flow free surface.

Case (b)
both sides inwarded w.r.t flow free surface.

Case (c)
one side outwarded and the other side inwarded w.r.t flow free surface.

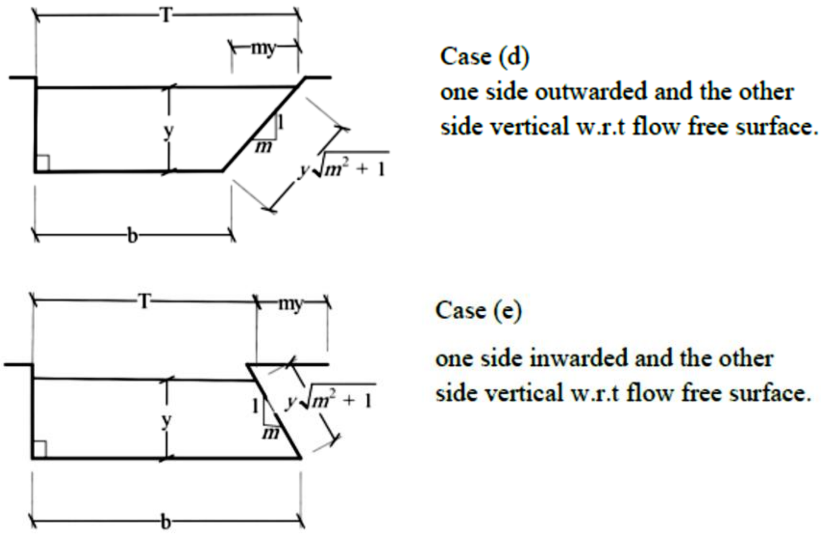


Figure 2: Open channel flow Sections

The five cases of possible cross-section shapes are carefully investigated to determine the best section conditions. The variables are y and b (T is function of y & b), P (witted perimeter) has to be minimum and b is expressed in terms of y and contacts (A & m's). The formulation and calculations for all cases are presented in Appendix A.

RESULTS AND CONCLUSIONS

A careful and deep analysis of all cases considered confirmed that for any flow cross-section shape which has a horizontal base width and two sides with different slopes the criteria for most efficient section will result when (i) channel width at free surface; known as top width T, equals the sum of the side slope lengths and (ii) the hydraulic radius is half the flow depth.

The first condition implies and contains the second condition and not vice versa.

This is a new concept in this area which allows using other shapes of flow sections as best sections rather than trapezoidal and rectangular sections.

Case (a) can be transformed into a symmetric trapezoid section if $m_1 = m_2 = m$, and the rectangular section results from any case considered when $m_1 = m_2 = m = 0$.

This new concept offers a new tool and breakthrough in determining best sections using graphical solutions for many combination of flow depths, base widths and side slopes which give the designer a much room of flexibility in determining best sections. A graphical method is presented in the following example, and other examples are given in Appendix B.

A. Given bed with width b and flow depth y, the method is as follow, Figure (3):

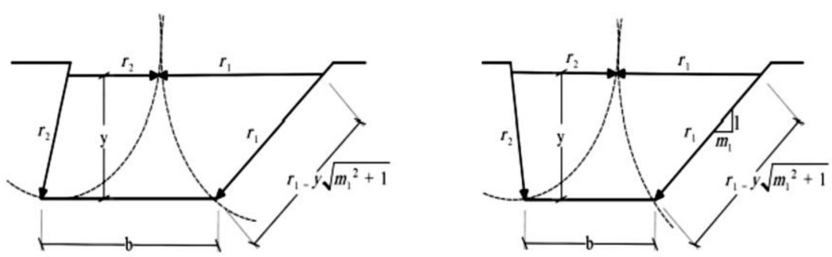


Figure 3: Graphical solution

1. Choose a reasonable scale for drawing.
2. Draw the bed width b.
3. Draw a parallel line at a distance y from channel bed.
4. At any arbitrary point on this line as a center draw an arc r_1 passes through one end of base width and cuts the top line at point a.
5. Draw another arc of radius r_2 which passes through the other end of the base width and through point a on the top width, these two arcs determine the side slopes m_1 and m_2. Indeed, any of the five cases considered might be a best section.

Other examples are shown in appendix B.

RECOMMENDATION

This work can be extended for determining best section of compound sections with horizontal and tilted base widths for the case of symmetric and non-symmetric shapes.

REFERENCES

[1] V. T. Chow, "Open-Channel Hydraulics". McGraw- HilL, New York, N.Y, (1951).

[2] F. M. Henderson, "Open Channel Flow". Macmillan, New York, N.Y, (1966).

[3] R.L. Daugherty and etal., "Fluid Mechanics with Engineering Applications". McGraw Hill, New York, N.Y, (1985)

Note. Few references were listed above and the reason behind that is the nature of this study, it’s a genuine study of an original work which could be a base as a reference for future studies.

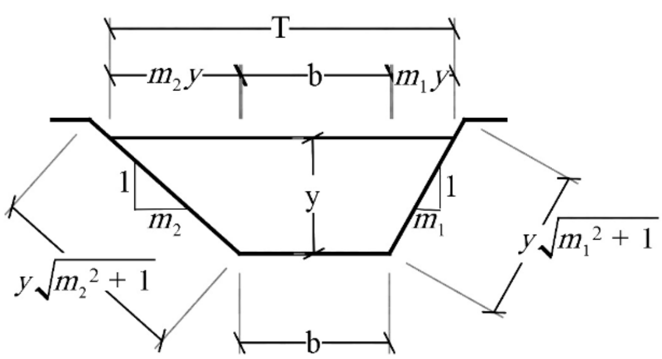
APPENDIX A

Mathematical derivation of most efficient section for different flow cross-section

Case (a) both sides out warded w.r.t. to flow free surface.

$$A = \left(\frac{m_1+b+m_2y+b}{2}\right)y$$

$$= [(m_1+m_2)y+2b]\frac{y}{2}$$



$$\frac{2A}{y} = (m_1+m_2)y+2b$$

$$\therefore b = \frac{2A}{2y} - \left[\frac{(m_1+m_2)y}{2}\right]$$

$$= \frac{A}{y} - \left(\frac{m_1+m_2}{2}\right)y$$

$$P = b + y\sqrt{m_1^2+1} + y\sqrt{m_2^2+1}$$

$$= \frac{A}{y} - \left(\frac{m_1+m_2}{2}\right)y + y\sqrt{m_1^2+1} + y\sqrt{m_2^2+1}$$

$$\frac{dP}{dy} = -\frac{A}{y^2} - \left(\frac{m_1 + m_2}{2}\right) + \sqrt{m_1^2 + 1} + \sqrt{m_2^2 + 1} = 0 \quad for \min. P$$

$$\frac{A}{y^2} = -\left(\frac{m_1 + m_2}{2}\right) + \sqrt{m_1^2 + 1} + \sqrt{m_2^2 + 1}$$

$$A = y^2 \left[-\left(\frac{m_1 + m_2}{2}\right) + \sqrt{m_1^2 + 1} + \sqrt{m_2^2 + 1}\right]$$

$$but\ A = [(m_1 + m_2)y + 2b]\frac{y}{2}$$

$$\therefore y^2 \left[-\left(\frac{m_1 + m_2}{2}\right) + \sqrt{m_1^2 + 1} + \sqrt{m_2^2 + 1}\right] = [(m_1 + m_2)y + 2b]\frac{y}{2}$$

$$-2y\left(\frac{m_1 + m_2}{2}\right) + 2y\left(\sqrt{m_1^2 + 1} + \sqrt{m_2^2 + 1}\right) = \left(\frac{m_1 + m_2}{2}\right)y + 2b$$

$$\therefore b + (m_1 + m_2)y = y\sqrt{m_1^2 + 1} + y\sqrt{m_2^2 + 1}$$

$$\therefore Top\ flow\ width = sum\ of\ side\ slope\ lengths$$

$$b = y\left[(m_1 + m_2) + \sqrt{m_1^2 + 1} + \sqrt{m_2^2 + 1}\right]$$

$$\therefore y = \frac{b}{\left[-(m_1 + m_2) + \sqrt{m_1^2 + 1} + \sqrt{m_2^2 + 1}\right]}$$

$$A = y^2 \left[-\left(\frac{m_1 + m_2}{2}\right) + \sqrt{m_1^2 + 1} + \sqrt{m_2^2 + 1}\right]$$

$$P = \frac{A}{y} - \left(\frac{m_1 + m_2}{2}\right)y + y\sqrt{m_1^2 + 1} + y\sqrt{m_2^2 + 1}$$

$$= \frac{y^2}{y}\left[-\left(\frac{m_1 + m_2}{2}\right) + \sqrt{m_1^2 + 1} + \sqrt{m_2^2 + 1}\right] - \left(\frac{m_1 + m_2}{2}\right)y + y\sqrt{m_1^2 + 1} + y\sqrt{m_2^2 + 1}$$

$$= -y\left(\frac{m_1 + m_2}{2}\right) + 2y\left(\sqrt{m_1^2 + 1} + \sqrt{m_2^2 + 1}\right) - \left(\frac{m_1 + m_2}{2}\right)y + y\left(\sqrt{m_1^2 + 1} + \sqrt{m_2^2 + 1}\right)$$

$$= 2y\left[\left(\frac{m_1 + m_2}{2}\right) + \sqrt{m_1^2 + 1} + \sqrt{m_2^2 + 1}\right]$$

$$R_h = \frac{A}{P} = \frac{y^2}{2y}\frac{\left[\left(\frac{m_1 + m_2}{2}\right) + \sqrt{m_1^2 + 1} + \sqrt{m_2^2 + 1}\right]}{\left[\left(\frac{m_1 + m_2}{2}\right) + \sqrt{m_1^2 + 1} + \sqrt{m_2^2 + 1}\right]} = \frac{y}{2}$$

$$\therefore Hydraulic\ radius\ (R_h) = half\ flow\ depth\ (y)$$

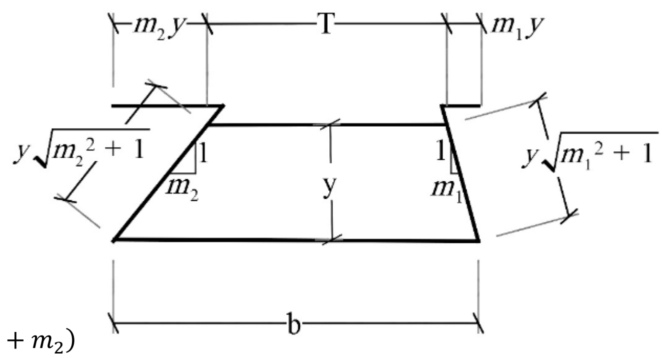
Case (b) both sides inwarded w.r.t. flow free surface.

$$T = b - m_1y - m_2y$$

$$= b - (m_1+m_2)y$$

$$A = \left(\frac{T+b}{2}\right)y$$

$$= \frac{y}{2}[b - y(m_1+m_2) + b]$$



$$A = \frac{y}{2}[2b - y(m_1+m_2)] = yb - \frac{y^2}{2}(m_1+m_2)$$

$$\therefore b = \tfrac{A}{y} + \tfrac{y}{2}(m_1+m_2), \qquad P = b + y\left(\sqrt{m_1^2+1} + \sqrt{m_2^2+1}\right)$$

$$P = \frac{A}{y} + \frac{y}{2}(m_1+m_2) + y\left(\sqrt{m_1^2+1} + \sqrt{m_2^2+1}\right)$$

$$\frac{dP}{dy} = -\frac{A}{y^2} + \frac{1}{2}(m_1+m_2) + \sqrt{m_1^2+1} + \sqrt{m_2^2+1} = 0 \quad for\ P_{min.}$$

$$\therefore A = y^2\left(\frac{m_1+m_2}{2} + \sqrt{m_1^2+1} + \sqrt{m_2^2+1}\right)$$

$$b = \frac{y^2}{y}\left(\frac{m_1+m_2}{2} + \sqrt{m_1^2+1} + \sqrt{m_2^2+1}\right) + \frac{y}{2}(m_1+m_2)$$

$$b = y\left[2\left(\frac{m_1+m_2}{2}\right)\right] + \left(\sqrt{m_1^2+1} + \sqrt{m_2^2+1}\right)y$$

$$b - y(m_1+m_2) = y\left(\sqrt{m_1^2+1} + \sqrt{m_2^2+1}\right)$$

$$Top\ width = Sum\ of\ side\ slopes\ length$$

$$y = \frac{b}{\sqrt{m_1^2+1} + \sqrt{m_2^2+1} + m_1 + m_2}$$

$$P = \frac{A}{y} + \frac{y}{2}(m_1+m_2) + y\left(\sqrt{m_1^2+1} + \sqrt{m_2^2+1}\right)$$

$$= \frac{y^2}{2}\left[\frac{m_1+m_2}{2} + \sqrt{m_1^2+1} + \sqrt{m_2^2+1}\right] + \frac{y}{2}(m_1+m_2) + y\left(\sqrt{m_1^2+1} + \sqrt{m_2^2+1}\right)$$

$$= 2y\left[\frac{m_1+m_2}{2} + \sqrt{m_1^2+1} + \sqrt{m_2^2+1}\right]$$

$$R_h = \frac{A}{P} = \frac{y^2\left[\frac{m_1+m_2}{2} + \sqrt{m_1^2+1} + \sqrt{m_2^2+1}\right]}{2y\left[\frac{m_1+m_2}{2} + \sqrt{m_1^2+1} + \sqrt{m_2^2+1}\right]} = \frac{y}{2}$$

$$\therefore Hydraulic\ radius\ (R_h) = half\ flow\ depth\ (y)$$

Case (c) one side in warded and the other side out warded w.r.t. flow free surface.

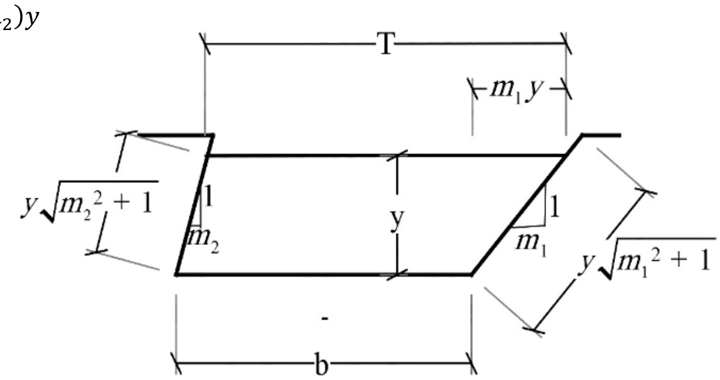
$$T = b + m_1y - m_2y \quad = b + (m_1 - m_2)y$$

$$A = \left(\frac{T+b}{2}\right)y = \left(\frac{b+y(m_1-m_2)+b}{2}\right)y$$

$$= \frac{2by}{2} + \frac{y^2}{2}(m_1 - m_2)$$

$$= by + \frac{y^2}{2}(m_1 - m_2)$$

$$\therefore b = \frac{A}{y} - \frac{y}{2}(m_1 - m_2)$$



$$P = b + y\left(\sqrt{m_1^2+1} + \sqrt{m_2^2+1}\right) = \frac{A}{y} - \frac{y}{2}(m_1 - m_2) + y\left(\sqrt{m_1^2+1} + \sqrt{m_2^2+1}\right)$$

$$\frac{dP}{dy} = -\frac{A}{y^2} + \sqrt{m_1^2+1} + \sqrt{m_2^2+1} - \frac{(m_1 - m_2)}{2} = 0 \quad for\, P_{min.}$$

$$A = y^2\left[\sqrt{m_1^2+1} + \sqrt{m_2^2+1} - \left(\frac{m_1 - m_2}{2}\right)\right]$$

$$b = \frac{A}{y} - \frac{y}{2}(m_1 - m_2) = \frac{y^2}{y}\left[\sqrt{m_1^2+1} + \sqrt{m_2^2+1} - \left(\frac{m_1 - m_2}{2}\right)\right] - \frac{y}{2}(m_1 - m_2)$$

$$b + 2\left(\frac{m_1 + m_2}{2}\right)y + y\sqrt{m_1^2+1} + y\sqrt{m_2^2+1}$$

$$b + y(m_1 - m_2) = y\sqrt{m_1^2+1} + y\sqrt{m_2^2+1}$$

$$Top\ width = Sum\ of\ side\ slopes\ length$$

$$b = \left(\sqrt{m_1^2+1} + \sqrt{m_2^2+1} - (m_1 - m_2)\right)$$

$$y = \frac{b}{\sqrt{m_1^2+1} + \sqrt{m_2^2+1} - m_1 + m_2}$$

$$A = y^2\left[\sqrt{m_1^2+1} + \sqrt{m_2^2+1} - \left(\frac{m_1 - m_2}{2}\right)\right]$$

$$P = b + y\left[\sqrt{m_1^2+1} + \sqrt{m_2^2+1}\right] = y\left[\sqrt{m_1^2+1} + \sqrt{m_2^2+1} - (m_1 - m_2)\right] + y\left(\sqrt{m_1^2+1} + \sqrt{m_2^2+1}\right)$$

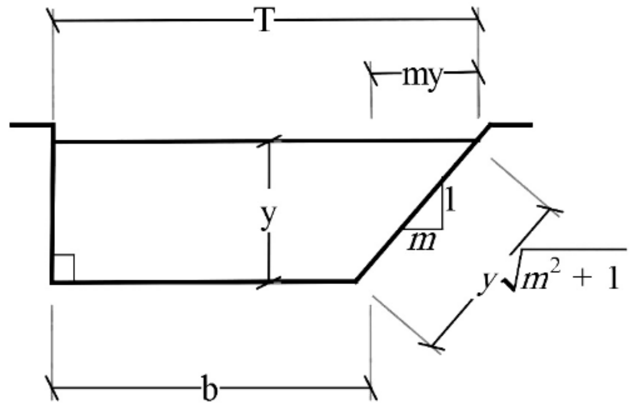
$$= 2y\left[\sqrt{m_1^2+1} + \sqrt{m_2^2+1} - \left(\frac{m_1 - m_2}{2}\right)\right]$$

$$R_h = \frac{A}{P} = \frac{y^2\left[\sqrt{m_1^2+1} + \sqrt{m_2^2+1} - \frac{m_1 - m_2}{2}\right]}{2y\left[\sqrt{m_1^2+1} + \sqrt{m_2^2+1} - \frac{m_1 - m_2}{2}\right]} = \frac{y}{2}$$

$\therefore Hydraulic\ radius\ (R_h) = half\ flow\ depth\ (y)$

Case (d) one side outewarded and the other side vertical w.r.t. flow free surface.

$T = b + my$



$A = y\left(\frac{T+b}{2}\right) = \left(\frac{b+my+b}{2}\right)y$

$\frac{A}{y} = b + \frac{my}{2}$

$b = \frac{A}{y} - \frac{my}{2}$

$P = y + b + y\sqrt{m^2+1}$

$\quad = y + \frac{A}{y} - \frac{my}{2} + y\sqrt{m^2+1}$

$\frac{dP}{dy} = 1 + \left(-\frac{A}{y^2}\right) - \frac{m}{2} + \sqrt{m^2+1} = 0 \quad for\ P_{min.}$

$\frac{A}{y^2} = 1 - \frac{m}{2} + \sqrt{m^2+1}$

$A = y^2\left[1 - \frac{m}{2} + \sqrt{m^2+1}\right]$

$b = \frac{y^2}{y}\left[1 - \frac{m}{2} + \sqrt{m^2+1}\right] - \frac{my}{2} = y - my + \sqrt{m^2+1} = y\left(1 - my + \sqrt{m^2+1}\right)$

$\therefore b + my = y + \sqrt{m^2+1}$

$Top\ width = Sum\ of\ side\ slopes\ length$

$\therefore b = y\left(1 - my + \sqrt{m^2+1}\right)$

$\therefore y = \frac{b}{\sqrt{m^2+1} + 1 - my}$

$P = y + \frac{y^2}{2}\left(1 - \frac{m}{2} + \sqrt{m^2+1}\right) - \frac{my}{2} + y\sqrt{m^2+1}$

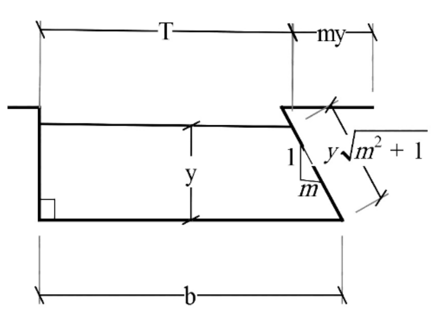
$\quad = y + y - \frac{my}{2} + y\sqrt{m^2+1} - \frac{my}{2} + y\sqrt{m^2+1} \ = 2y\left[1 + \sqrt{m^2+1} - \frac{m}{2}\right]$

$R_h = \frac{A}{P} = \frac{y^2\left[1 + \sqrt{m^2+1} - \frac{m}{2}\right]}{2y\left[1 + \sqrt{m^2+1} - \frac{m}{2}\right]} = \frac{y}{2}$

$\therefore Hydraulic\ radius\ (R_h) = half\ flow\ depth\ (y)$

Case (e) one side in warded and the other side vertical w.r.t. flow free surface.

$T = b - my$



$A = \left(\frac{T+b}{2}\right)y = \left(\frac{b-my+b}{2}\right)y \ = b - \frac{my}{2}$

$\frac{A}{y} = b - \frac{my}{2}$

$b = \frac{A}{y} + \frac{my}{2}$

$$P = y + b + y\sqrt{m^2+1} = y + \frac{A}{y} + \frac{my}{2} + y\sqrt{m^2+1}$$

$$\frac{dP}{dy} = 1 + \left(-\frac{A}{y^2}\right) + \frac{m}{2} + \sqrt{m^2+1} = 0 \quad for\ P_{min.}$$

$$\frac{A}{y^2} = 1 + \frac{m}{2} + \sqrt{m^2+1}$$

$$A = y^2\left[1 + \frac{m}{2} + \sqrt{m^2+1}\right]$$

$$b = \frac{A}{y} + \frac{my}{2} = \frac{y^2}{y}\left[1 + \frac{m}{2} + \sqrt{m^2+1}\right] + \frac{my}{2}$$

$$= y + my + y\sqrt{m^2+1}$$

$$b - my = y + y\sqrt{m^2+1}$$

$$Top\ width = Sum\ of\ side\ slopes\ length$$

$$b = y\left(1 + my + \sqrt{m^2+1}\right)$$

$$\therefore y = \frac{b}{1 + m + \sqrt{m^2+1}}$$

$$P = y + b + y\sqrt{m^2+1} = y + y\left(1 + m + \sqrt{m^2+1}\right) + y\sqrt{m^2+1}$$

$$= 2y + \left(\frac{my}{2}\right)^2 + 2y\sqrt{m^2+1}$$

$$= \left(1 + \frac{m}{2} + \sqrt{m^2+1}\right)2y$$

$$R_h = \frac{A}{P} = \frac{y^2\left[1 + \frac{m}{2} + \sqrt{m^2+1}\right]}{2y\left[1 + \frac{m}{2} + \sqrt{m^2+1}\right]} = \frac{y}{2}$$

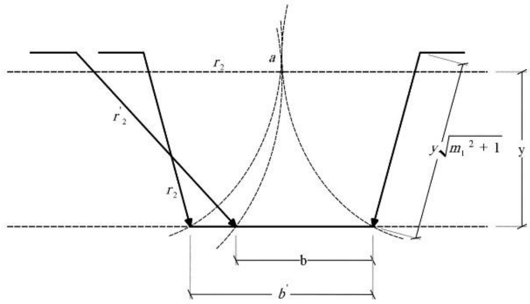
$$\therefore Hydraulic\ radius\ (R_h) = half\ flow\ depth\ (y)$$

APPENDIX B

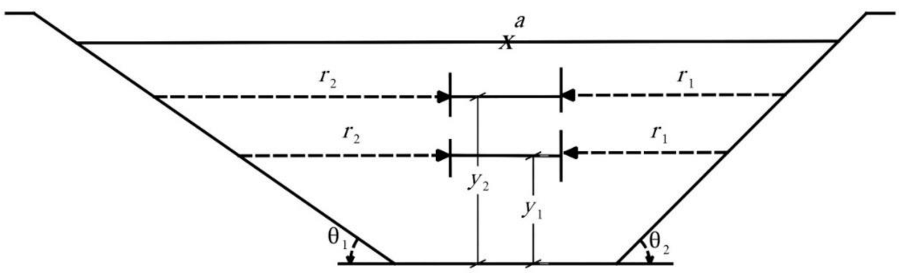
Determination of most efficient section; graphical solution

Examples:

a. Given side slope m_1, and depth y, the procedure is as follows:
   i. Choose a reasonable scale for drawing.
   ii. Draw two parallel and horizontal lines with y distance a part.
   iii. At a point on the lower line draw the channel side with the given slope m_1, to intersect top line at a point.
   iv. From this point as a center draw an arc of radius $r_1 = y\sqrt{m_1^2+1}$ to cut top line at a.
   v. At another point on top width as a center draw another arc of radius r_2 which passes through a and passes through lower line.
   vi. $r_2 = y\sqrt{m_2^2+1}$, hence m_2 and the channel other side is determined, this method can give three cases.



b. Given base width b and slope m_1 & m_2:
   i. Choose a reasonable scale for drawing.
   ii. Draw the channel with its base width b and two sides.
   iii. Choose assumed flow depth y and draw a parallel line to the base width with y distance a part, this determines $r_1 = y\sqrt{m_1^2 + 1}$ & $r_2 = y\sqrt{m_2^2 + 1}$.
   iv. At the two ends of the top line as centers draw two arcs with r_1 and r_2 radii.
   v. If the assumed depth satisfies the best section condition, then r_1 and r_2 should met at point a.
   vi. If not, then adjust y as required, this method can give seven possible cases.



c. Given flow depth y it would be possible to propose best sections for different values of m's and b's.

d. Given one side slope m_1 it would be possible to have different best sections for combinations of m_2's and y's.