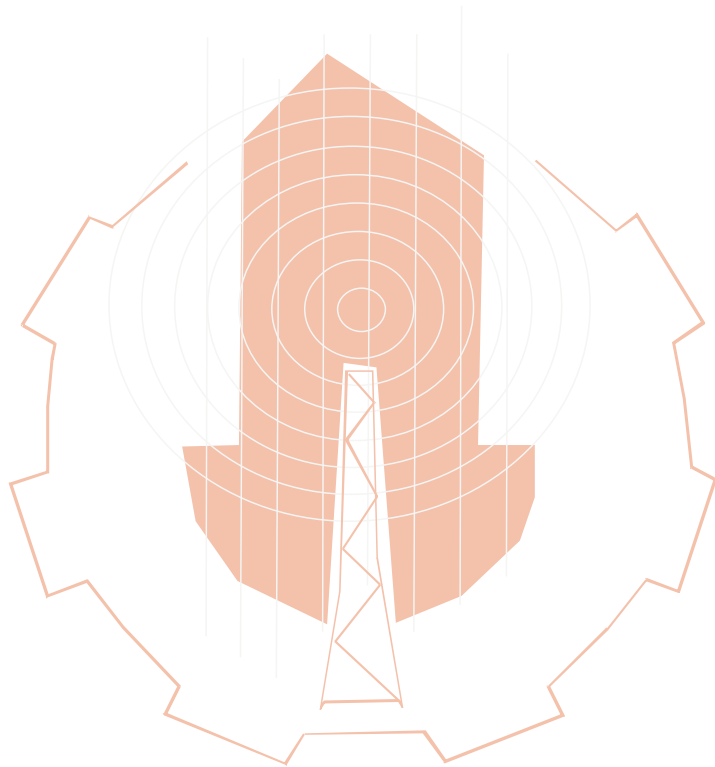




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# ANALYSIS AND ADJUSTMENT OF LIBYAN GEODETIC REFERENCE DATUM LGD2006

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## المخلص

لعب التطور العلمي الحديث في التقنية دوراً كبيراً في جميع المجالات الهندسية ومنها المساحة، حيث أصبحت البرامج المساحية بشتى أنواعها ذات قدرة عالية في تسهيل العمل واختصار الوقت والحصول على الدقة العالية.

نظم الإحداثيات الجغرافية والتربيعية الفراغية المستخدمة في عملية التحويل بين المراجع البيضاوية من نظام إلى آخر هي الحل الأمثل في الانتقال بين المراجع. وتنقسم إلى طرق عادية وطرق دقيقة والتي من خلالها تتم المراقبة الدقيقة لحركة القشرة الأرضية في تغيير عناصر الشكل البيضاوي WGS84 وتسمى International Terrestrial Reference Frame (ITRF) (الإطار المرجعي الدولي للأرض)، وهذا الأخير هو نقطة البحث، حيث قامت مصلحة المساحة الليبية لسنة 2006 م بوضع مرجع إسناد خاص بها تحت اسم المرجع الجيوديسي الليبي (LGD2006.3822) لسنة 2006 م.

تعرض هذه الورقة تحليل بيانات مرجع الإسناد (LGD2006.3822) ومدى دقتها من خلال المحطات التي تم استخدامها لربط هذا المرجع، كذلك إظهار عناصر التحويل للتغير الزمني بين المرجع العالمي ITRF2000 وLGD2006 وحساب أخطاء الرصد بين النقاط المحلية الرئيسية الخمس بمحطات دائمة الرصد وعددها إحدى عشرة محطة معتمدة من الجمعية الأمريكية العلمية المساحية (والمستخدمة من مصلحة المساحة الليبية لسنة 2000 م)؛ وقد بينت الدراسة بأن عدد المحطات المستخدمة في الربط غير متناسقة من حيث المسافات مما أثر سلباً على الانحراف المعياري لبيانات المحطات المرصودة بالنسبة لمتوسطة الأخطاء التربيعية (RMS).

## ABSTRACT

The modern scientific development in technology plays a big role in all engineering fields, including geodetic and systems of coordinate, surveying programs important for facilitating works and minimize the observation time with high accuracy and less cost.

The geographical coordinate, the geocentric coordinate and the triangular coordinate systems used in the processing of transformation between datum references from one system to another. It was given the ideal solution for observing International Terrestrial Reference Frame (ITRF) and the changing in parameters of WGS84, which is the research objective of this work, where the Survey Department of Libya "SDL" created a new datum reference Libyan geodetic datum LGD2006.3822.

This paper presents an analysis of the LGD2006 and its accuracy through connecting by international fixed stations, as well as showing the transformation parameters ITRF between the global reference ITRF2000 G1150 and also calculating the errors of the five fixed stations; based on the eleven international fixed stations which

approved by the scientific society of American Survey for the year 2000 which is used from SDL.

This paper presents the international stations which used in LGD2006.3822 project in non-homogenous “on distances meaning” that has negative effect on root mean square error (RMS).

**KEYWORDS:** ITRF; LGD2006; Transformation; Geocentric Coordinate; SDL.

## INTRODUCTION

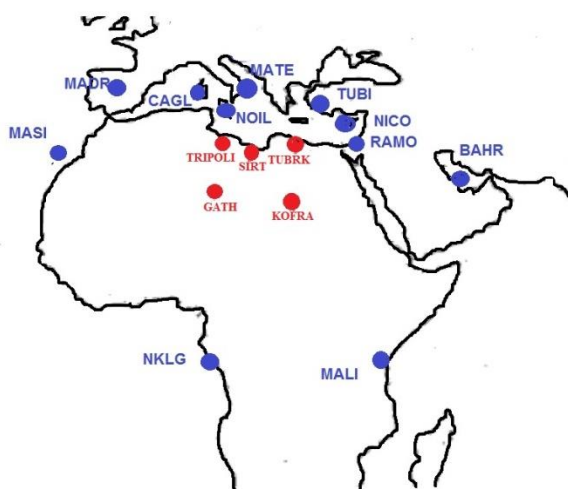
There are many methods used to transform the coordinate systems between two references datum, one of these methods Burch-Wolf model, which depend on their transformations to the geocentric coordinate, also Moldinski method uses transformation with mathematical functions most of the use parameters of Helmert; while there are other methods called polynomial methods use their own approximate equations, which are suitable only for the area designated for it.

The Libyan reference LGD2006 for the epoch 2006 were constructed on five main fixed points distributed on (Tripoli, Tubrk, Kofra, Sirt and Gath) these coordinate was computed using the eleven international fixed stations that distributed in Europe- Asia – Africa; An extra 61 stations had been constructed and computed based on the five fixed stations using Hayford international reference 1924. All stations coordinates transformed from ITRF2000-WGS84 “G1150” generation for the year 2000.

The distances between the five fixed stations and the eleven international stations were computed using geodetic measurement problems based on Bessel method [1].

## METHODOLOGY AND CASE STUDY

The study area includes five fixed stations approved by SDL (Survey Department of Libya), as well as eleven international fixed stations distributed in Southern Europe, West Asia and Central Africa (Figure 1), and its coordinate are listed in Tables (1 and 2) [2].



**Figure 1: Illustrate positioning of stations "blue- international, red- local fixed stations" [2,3]**

**Table1: illustrate Data of five local fixed stations [2]**

Point	geographic coordinate WGS84 ITRF2000	2006 geographic coordinate LGD 2006.3822 Epoch-ITRF2000
Tubrk	$\varphi=31^{\circ} 51' 46.40236''$ N	$\varphi=31^{\circ} 51' 45.100397''$ N
	$\lambda=24^{\circ} 01' 20.85196''$ E	$\lambda=24^{\circ} 01' 21.442316''$ E
	H <sub>GEO</sub> =179.5618m	H <sub>GEO</sub> =154.983m
kofra	$\varphi=24^{\circ} 05' 51.65568''$ N	$\varphi=24^{\circ} 05' 50.824125''$ N
	$\lambda=23^{\circ} 16' 51.59393''$ E	$\lambda=23^{\circ} 16' 52.251036''$ E
	H <sub>GEO</sub> =448.7385m	H <sub>GEO</sub> =428.375m
Gath	$\varphi=24^{\circ} 58' 10.78608''$ N	$\varphi=24^{\circ} 58' 10.049278''$ N
	$\lambda=10^{\circ} 10' 47.41005''$ E	$\lambda=10^{\circ} 10' 49.952198''$ E
	H <sub>GEO</sub> =742.411m	H <sub>GEO</sub> =712.293m
Sirt	$\varphi=31^{\circ} 09' 57.37869''$ N	$\varphi=31^{\circ} 09' 56.188813''$ N
	$\lambda=16^{\circ} 42' 03.23437''$ E	$\lambda=16^{\circ} 42' 04.946779''$ E
	H <sub>GEO</sub> =83.6224m	H <sub>GEO</sub> =56.190m
Tripoli	$\varphi=32^{\circ} 51' 30.75676''$ N	$\varphi=32^{\circ} 51' 29.513385''$ N
	$\lambda=13^{\circ} 05' 26.12293''$ E	$\lambda=13^{\circ} 05' 28.423534''$ E
	H <sub>GEO</sub> =69.696m	H <sub>GEO</sub> =38.346m

**Table2: illustrate Data of eleven international fixed stations [2]**

geographic coordinate WGS84 G1150 ITRF2000			
BAHR	$\varphi=35^{\circ} 08' 27.55100''$ N	NICO	$\varphi=26^{\circ} 12' 32.90958''$ N
	$\lambda=33^{\circ} 23' 47.20641''$ E		$\lambda=50^{\circ} 36' 29.32639''$ E
	H <sub>GEO</sub> =190.0032m		H <sub>GEO</sub> =-17.090m
CAGL	$\varphi=27^{\circ} 45' 49.46896''$ N	MASI	$\varphi=39^{\circ} 08' 09.28305''$ N
	$\lambda=15^{\circ} 37' 59.79180''$ W		$\lambda=08^{\circ} 58' 21.91040''$ E
	H <sub>GEO</sub> =197.1414m		H <sub>GEO</sub> =238.3496m
MADR	$\varphi=00^{\circ} 21' 14.06350''$ N	NKLK	$\varphi=40^{\circ} 25' 44.98343''$ N
	$\lambda=09^{\circ} 40' 19.65069''$ E		$\lambda=04^{\circ} 14' 58.77158''$ W
	H <sub>GEO</sub> =31.50181m		H <sub>GEO</sub> =829.4406m
MALI	$\varphi=36^{\circ} 52' 33.04034''$ N	NOTL	$\varphi=02^{\circ} 59' 45.27975''$ S
	$\lambda=14^{\circ} 59' 23.23522''$ E		$\lambda=40^{\circ} 11' 39.82412''$ E
	H <sub>GEO</sub> =126.3268m		H <sub>GEO</sub> =-23.3550m
MATE	$\varphi=30^{\circ} 35' 51.38023''$ N	RAMO	$\varphi=40^{\circ} 38' 56.87436''$ N
	$\lambda=34^{\circ} 45' 47.30300''$ E		$\lambda=16^{\circ} 42' 16.05521''$ E
	H <sub>GEO</sub> =886.8166m		H <sub>GEO</sub> =535.6333m
TUBI	$\varphi=40^{\circ} 47' 12.21005''$ N		
	$\lambda=29^{\circ} 27' 02.46180''$ E		
	H <sub>GEO</sub> =220.3154m		

**Methods of Coordinate’s Transformation**

The coordinate transformation can be done by two ways: geocentric coordinate and mathematics functions of ellipsoid.

**Method of Burch-Wolf**

This method is very simple to transform the coordinate form datum to other datum [4].

- Obtain geocentric coordinate WGS84 from geographic coordinates by using equation (1):

$$X = (N + H)\cos\varphi\cos\lambda, Y = (N + H)\cos\varphi\sin\lambda, Z = ((1 - e^2)N + H)\sin\varphi \quad (1)$$

- Transformation from WGS84ITRF2000 to LGD2006.3822

$$\begin{pmatrix} X_{2006} \\ Y_{2006} \\ Z_{2006} \end{pmatrix} = \begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix} + (2006.3822 - 2000) \begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix} + \begin{pmatrix} 1 & \omega_z & -\omega_y \\ -\omega_z & 1 & \omega_x \\ \omega_y & -\omega_x & 1 \end{pmatrix} \begin{pmatrix} X_{2000} \\ Y_{2000} \\ Z_{2000} \end{pmatrix} \quad (2)$$

- Transformation from geocentric coordinate LGD2006.3822 to geographic coordinates LGD2006.3822 without iteration [4];

$$\left. \begin{aligned} \lambda_a &= \tan^{-1} \frac{Y}{X}; \quad Q = \sqrt{X^2 + Y^2}; \quad B = \tan^{-1} \frac{Z}{Q(1-e^2)} \\ H_{GEO} &= R - \frac{b}{\sqrt{1-e^2\left(\frac{Q}{R}\right)^2}} \quad \varphi = \operatorname{atan} \left( \left( 1 - \frac{H_{Geo} \cdot e^2 \tan(B)}{Z \cdot \sqrt{1+(\tan(B))^2}} \right) \tan(B) \right) \end{aligned} \right\} \quad (3)$$

### Method of Moldinski

This method presents transformation coordinate directly from geographic coordinates WGS84- G1150 to geographic coordinates LGD2006.3822, with accuracy of 0.0001" It needs to be repeated once [4].

$$\left. \begin{aligned} \varphi_B &= \varphi_A + \Delta\varphi, \quad \lambda_B = \lambda_A + \Delta\lambda, \quad H_B = H_A + \Delta H \\ \Delta\varphi &= \frac{1}{(M+H)} \left[ e^2 \frac{N}{a} \sin\varphi \cos\varphi \Delta a + \left( 1 + \frac{N^2}{a^2} \right) N \sin\varphi \cos\varphi \frac{\Delta e^2}{2} + \Delta Z \cos\varphi - \right. \\ &\quad \left. - (\Delta X \cos\lambda + \Delta Y \sin\lambda) \sin\varphi \right] - (\omega_x \sin\lambda - \omega_y \cos\lambda)(1 + e^2 \cos 2\varphi) - m e^2 \sin\varphi \cos\varphi \\ \Delta\lambda &= \frac{1}{(N+H) \cos\varphi} (\Delta Y \cos\lambda - \Delta X \sin\lambda) + (1 + e^2) \tan\varphi (\omega_y \sin\lambda + \omega_x \cos\lambda) - \omega_z \\ \Delta H &= \frac{\Delta e^2}{2} N (\sin\varphi)^2 - \Delta a \frac{a}{N} + (\Delta X \cos\lambda + \Delta Y \sin\lambda) \cos\varphi + \Delta Z \sin\varphi - \\ &\quad - e^2 N \sin\varphi \cos\varphi (\omega_x \sin\lambda - \omega_y \cos\lambda) + \left( \frac{a^2}{N} + H \right) m \\ \varphi_B &= \frac{\varphi_A + (\varphi_A + \Delta\varphi)}{2}, \quad \lambda_B = \frac{\lambda_A + (\lambda_A + \Delta\lambda)}{2}, \quad H_B = \frac{H_A + (H_A + \Delta H)}{2} \end{aligned} \right\} \quad (4)$$

### Errors Determination of Coordinate Transformation

These Errors are important to check the accuracy; it can be determined by differential equation (1) as following [4,5]:

$$\left. \begin{aligned} X &= f(\varphi, \lambda, N), \quad Y = f(\varphi, \lambda, N), \quad Z = f(\varphi, \lambda, N), \dots, \quad N = \frac{a}{\sqrt{1-e^2(\sin\varphi)^2}} \\ \begin{pmatrix} \sigma_x^2 \\ \sigma_y^2 \\ \sigma_z^2 \end{pmatrix} &= \begin{pmatrix} ((M+H) \sin\varphi \cos\lambda)^2 & ((N+H) \cos\varphi \sin\lambda)^2 & (\cos\varphi \cos\lambda)^2 \\ ((M+H) \sin\varphi \sin\lambda)^2 & ((N+H) \cos\varphi \cos\lambda)^2 & (\cos\varphi \sin\lambda)^2 \\ ((M+H) \cos\varphi)^2 & 0 & (\sin\varphi)^2 \end{pmatrix} \begin{pmatrix} \left( \frac{\sigma_\varphi \pi}{64800} \right)^2 \\ \left( \frac{\sigma_\lambda \pi}{64800} \right)^2 \\ \sigma_H^2 \end{pmatrix} \end{aligned} \right\} \quad (5)$$

Where:  $\sigma_x^2, \sigma_y^2, \sigma_z^2$  values of errors in meters

### Methods of Geodesy Problems for Measuring Distances

The best method for measuring distances is Bessel method with high accuracy for long distance about 20000 km, it has been used for measuring distances between five fixed station and eleven international stations for determining Signal delay time.

### The Effect Plate Tectonics on Transformation Coordinate

The plate mathematical model is defined by the different between epochs, in addition angular rotation matrix as [6]:

$$\begin{pmatrix} d_{tx} \\ d_{ty} \\ d_{tz} \end{pmatrix} = (\text{from epoch} - \text{to epoch}) \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & -\omega_x & 0 \end{pmatrix} \begin{pmatrix} x_{2000} \\ y_{2000} \\ z_{2000} \end{pmatrix} \quad (6)$$

The values of the tectonic plate should be added to equation (5).

### DATA ANALYSIS

Some programs have been written in this study to do data analysis; first obtaining rate of transformation parameters between WGS84-ITRF2000 and LGD2006, also geocentric coordinates LGD2006, because all data were not presented in the project report (Survey Department of Libya); this research calculated and presents geocentric coordinates for five fixed stations illustrate in Table (3).

**Table 3: illustrates Data of five local fixed stations**

Trans. Parameters of rate time ITRF2000 to epoch2006.3822 T <sub>x</sub> =-0.00164 m T <sub>y</sub> =-0.00056 m T <sub>z</sub> =0.00694 m Trans. Parameters WGS84-ITRF20000 to LGD2006.3822 Hayford m dX=208.4058m dY=109.8777m dZ=2.5764m						
point	Geocentric coord ITRF2000		Errors m	Geocentric coordLGD2006		Errors m
Tubrk	X	4952608.7783	0.0020	X	4952817.1736	0.0020
	Y	2207370.0589	0.0025	Y	2207479.9330	0.0025
	Z	3347623.2309	0.0027	Z	3347625.8516	0.0027
Kofra	X	5351584.7336	0.0016	X	5351793.1289	0.0016
	Y	2302652.8135	0.0028	Y	2302762.6876	0.0028
	Z	2588346.6849	0.0028	Z	2588349.3056	0.0028
Gath	X	5695020.8949	0.0016	X	5695229.2902	0.0016
	Y	1022627.0132	0.0028	Y	1022736.8873	0.0028
	Z	2676341.8105	0.0028	Z	2676344.4312	0.0028
Sirt	X	5232142.3959	0.0019	X	5232350.7913	0.0019
	Y	1569807.3741	0.0026	Y	1569917.2482	0.0026
	Z	3281693.0009	0.0026	Z	3281695.6216	0.0026
Tripoli	X	5223692.4518	0.0019	X	5223900.8472	0.0019
	Y	1214686.9438	0.0026	Y	1214796.8180	0.0026
	Z	3440828.8801	0.0026	Z	3440831.5008	0.0026

The Table (3) shows geographic coordinate transformation systems WGS84G1150 to geocentric coordinate for ITRF2000 and LGD2006.3822, also transformation parameters between them, all results are not in the description cards of SDL.

Uses of geodetic Problems for measuring distances between station of Tripoli and eleven international stations" example", all results are shown in Table (4).

**Table 4: illustrates determine Signal delay time.**

Positioning	Distances m	Azimuth angles	delay time second	Errors m
Tripoli-BAHR	3673281.112	110°34'35.4018"	0.01227	0.3783
CAGL- Tripoli	788313.904	330°48'54.4416"	0.00263	0.0812
MADR- Tripoli	1759101.073	293°02'07.1077"	0.00587	0.1812
MASI- Tripoli	2797515.699	251°17'47.7766"	0.00933	0.2881
MALI- Tripoli	4878724.207	146°36'29.2872"	0.01628	0.5025
MATE- Tripoli	922575.649	21°34'24.7178"	0.00308	0.095
NICO- Tripoli	1889606.676	88°06'51.2150"	0.00631	0.1946
NKLG- Tripoli	3615740.754	185°21'31.8954"	0.01206	0.3724
NOTL- Tripoli	478276.775	21°50'13.5823"	0.001596	0.0493
RAMO –Tripoli	2055729.302	102°36'39.7735"	0.0069	0.2117
TUBI- Tripoli	1700063.356	64°02'52.6117"	0.00567	0.1751

Also Table (4) illustrate the errors resulting between point of Tripoli and other points "eleven", Where the error is large whenever the greater the distance; while the average error calculated from SDL by program Leica geo-office 0.230 m.

## CONCLUSIONS

- This research resolved transformation parameters for ITRF between WGS84 G1150 and LGD2006.3822 with accuracy nearly 0.001 m;
- This research illustrated geocentric coordinates for ITRF2000 and LGD2006.3822;
- This research presented the international stations which used in LGD2006.3822 project are not homogenous “on distances meaning” that has negative effected on RMS.
- To increase accuracy, it is better to use only the stations nearby "CAGL, MADR, MATE, NOTL, TUBI".
- This research resolved all problems, and every user can transform the coordinates from ITRF2000 to LGD2006.3822 easily without using any expensive program.

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