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التحليل العددى لتدفق على أسطح مائلة (أسفين)

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قسم الهندسة البحرية ـ كلية الهندسة ـ جامعة الفاتح

NUMERICAL SOLUTION FOR FLOW PAST-WEDGES

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ملخص

منحنى السرعة واجهاد القص ومعامل السحب تتأثر بزاوية الميل وبالتالى بانخفاض الضغط لتدفق مستقر، غير قابل للانضغاط والانسياب على الأسطح المائلة.

وفى هذه الورقة نستعمل طريقة رانج ـ كوتا التكاملية لحل معادلة فالكنار ـ سكان عددياً وهذه المعادلة هى معادلة تفاضلية عادية من الدرجة الثالثة حيث لا يوجد لها حل تحليلى معروف. وبحل هذه المعادلة عددياً يمكن أيضاً ايجاد تأثير انخفاض الضغط على سمك الطبقة الحدية وسمك الازاحة وسمك كمية الحركة.

Abstract:

A computer program is run to solve Falkner-Skan equation for steady, incopressible, laminar flow past wedges. The program integrates the Falkner-Skan equation using a fourth order Runge-Kutta integration scheme. The Falkner-Skan equation is a third order non-linear ordinary differential equation for which no analytical solution exists. It is usually associated with a two-point an asymptotic boundary value problem. The velocity distribution, the wall-shear stress and the drag-coeffecient are evaluated. Finally the displacement, momentum and boundary layer thicknesses are obtained.

Introduction:

A complete solutions for flow past wedges will be found. The paper will deal with some exact solutions of the boundary layer equations. A solution will be considered exact when it is a complete solution of the boundary layer equations, irrespective of whether it is obtained analytically or by numerical methods.

Falkner-Skan equations:

It is known that similarity solutions exist when the velocity of the potential flow is proportional to a power of the lenght, x, measured from the stagnation point, i e for:

U (x) = C. × 1M
M = B/ (2-B) or B = 2M/ (M+1)
U (x) = potential flow velacity.
B = Falkner-Skan parameter.
C = constant.

Since the flow outside the boundary layer is inviscid flow, Euler's equation can be used to relate the potential flow velocity to pressure gradient. The differentiation of this equation with respect to x gives:

dp/dx = − d.U (x). dU(x)/ dx

where d is the fluid density.
Since U = C. x 1M, it can be showm that:
dp/dx = − (d.M/x). U 12.

The physical meaning of B is indicated in fig (1). The value of «M» specifies the inviscid pressure gradient. For instance, B = 0 and M = 0, for flow over a flat plate with zero pressure gradient. Thus U = C, a constant free-stream velocity. For stagnation flow, B = 1.0 and M = 1.0. This requires that the free-stream velocity be given by U (x) = C.x.

The following ordinary differential equation and its associated boundary conditions is called the Falkner-

Skan equation for which numerical solution to be obtained by using a fourth-order Runge-Kutta integration scheme

The governing differential equation:

$F''' + F.F'' + B\,(1-F\,12) = 0.$

Boundary conditions:

$F\,(E) = 0$ at $E = 0$
$F'\,(E) = 0$ at $E = 0$
$F'\,(E) = 1.0$ as E approaching infinity

Where: $E = y.\ SQR\ [(M+1).\ \ U/2)u\ \ x] =$ dimensionless − distance
$F\ (E) = S/SQR\ [2U\ \ U\ \ x/\ (M+1)] =$ dimensionless − stream function
U = kinematic viscosity
S = stream function
F′ (E) = the first derivative of F (E)
F″ (E) = the second derivative of F (E)
u = x − component velocity
U = free − stream velocity

WALL SHEAR-STRESS AND DRAG COEFFICIENT:

$T\,(x, 0) = udu/dy = uU.\ SQR\ [(M+1)\ U/2U\ x\,]\ F''\,(0)$
it can be shown that:

$T\,(x, o) = uU.\ SQR\ [(M+1)\ /2]\ [SQR\ (Rex)\ /x]\ F''\,(0)$
Also it can be showm that:

$T = 2T\,(x, o)$ at $x = L$
T (x, o) = local shear-stress
T = Average shear stress over the entire length.
The value of F″ (0) will be evaluated numerically.
u = dynamic viscosity
$D\,(x) = 2T\,(x, o)\ /d.\ U12$ = Local skin friction
$D\,(x) = 2.\ SQR\ [(M+1)/\ 2].\ F''\,(0)\ .1/SAR\ (Rex)$

Also:
$D = 2.\ D\,(x)$ at $x = L$
D (x) = local skin-friction
D = average skin-friction over the entire length
Rex = U. x/U local Reynolds's number

DISPLACEMENT AND MOMENTUM THICKNESSES:

The displacement thickness may be interpreted as the decrease due to viscous effects, with respect to the equivalent inviscid flow, in the mass low rate between the surface and a stream line at large distance from the surface. In terms of the defined similarity parameters the displacement thickness «DT» is defined as follows for incompressible flow:

$$DT\,(x) = \int_0^{\infty} (1-u/U)\,dy = SQR\ [2.\ \nu\ x/(M+1)\ U]\ \int_0^{\infty} (1-F')\,dE.$$

$$\text{Let } I1 = \int_0^{\infty} (1-F')\,dE$$

The value of I1 will be evaluated numerically.

$DT\,(x) = I1.\ SQR\ [2/\ (M+1].\ x/SQR\ (Rex)$
$DT = (2/3).\ DT\,(x)$ at $x = L$
DT (x) = local displacement thickness.
DT = Average displacement thickness over the entire length.

The momentun thickness is a measure of the decrease in momentun of the mass in the boundary-layer with respect to the value it would have in the equivalent inviscid flow field. The momentum thickness «MT» for incompressible flow is given by:

$$MT\,(x) = \int_0^{\infty} (u/U).\ (1-u/U)\,dy = SQR\ [2\nu\ x]\ /U\ (M+1)\ \int_0^{\infty} F'\,(1-F')\,dE.$$

$$\text{Let } I2 = \int_0^{\infty} F'\,(1-F')\,dE.$$

The value of I2 will be evaluated numerically.
$MT\,(x) = I2.\ SQR\ [2/\ (M+1)].\ x/SQR\ (Rex).$
$MT = (2/3).\ MT\,(x)$ at $x = L$

MT (x) = local momentum displacement
MT = Average momentum displacement

BOUNDARY LAYER THICKNESS:

It is desired to define the boundary layer thickness as that distance for which:
$u\,(x) = 0.99U\,(x).$

Results:

Numerical example: For «B» = 0.15
M = 0.8108
F″ (0) = 0.63860
I1 = 1.028607640
I2 = 0.421143753

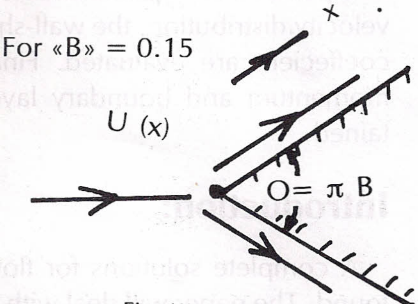

Fig. (1)

Wall shear-stress:
T (x, o) = 0.4695 U. SQR (Rex) /x
T = 0.9390 U. SQR (Rel/L)
Skin-friction:
D (x) = 0.9390, SQR (Rex)
D = 1.8780. SQR (Rel)

Displacement thickness:
DT (x) = 1.3911. x/SQR (Rex)
DT = 0.9327.x/SQR (Rel)

Momentum thickness:
MT (x) = 0.4553.x/SQR (Rex)
MT = 0.3035. L/SQR (Rel)
Boundary layer thickness: FIG (1)
F′ (E) = u/U = 0.99 at E = 3.18
Set: y = BT and E = 3.18, the result is:

BT = 4.33.x/SQR (Rex).
Rel = Reynolds number at x = L.

TABLE OF RESULTS (I):

B	A(x)	G(x)	J(x)
0.0	.3321	.6642	4.92
0.15	.4695	.9390	4.33
1/3	.6213	1.2426	3.79
1/2	.7575	1.5150	3.28
2/3	.8997	1.7994	3.01
1.0	1.2326	2.4652	2.38
4/3	1.7150	3.4300	1.80
3/2	2.0891	4.1782	1.52
5/3	2.6857	5.3714	1.20

Where:
A (x) = [T (x, o)/ uU (x)]. x/ SQR (Rex).
G (x) = D (x). SQR (Rex).
J (x) = [BT (x)/ x]. SQR (Rex).
Note: T = 2T (x, o) at x = L
D = 2D (x, o) at x = L

TABLE OF RESULTS (II):

B	K(x)	N(x)	Q(x)
0.0	1.7208	0.6641	2.59
0.15	1.3991	0.5728	2.44
1/3	1.1492	0.4894	2.35
1/2	0.9854	0.4290	2.30
2/3	0.8547	0.3779	2.26
1.0	0.6479	0.2923	2.22
4/3	0.4765	0.2177	2.19
3/2	0.3945	0.1811	2.18
5/3	0.3091	0.1425	2.17

Where:
K(x) = [DT (x)/x]. SQR (Rex).
N(x) = [MT (x)/x]. SQR (Rex).
Q(x) = DT (x) /MT (x).

Note: DT = (2/3). DT (x) at x = L
MT = :2/3). MT (x) at x = L

Conclusions:

By obtaining solutions for various values of «B», o ≤ B < 2.0, the effect of various pressure gradient on wall shear-stress, skin-friction, displacement, momentum and boundary layer thicknesses are found. The solution gives the complete profiles of F, F′ and F″ as a function of «E» through the boundary layer.

It is found that, the wall shear-stress and the skin-friction increase with the increasing pressure gradient, while displacement, momentum and boundary-layer thicknesses are decreasing. The numerical values which are obtained using fourth order Runge-Kutta integration scheme, found to be very close to those found by other numerical methods.

References:

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Appendix:

FOR «B» = 0.0

E	F″	F′	F
0.0	0.4696	0.0000	0.0000
0.2	0.4693	0.0939	0.0094
0.4	0.4673	0.1876	0.0375
0.6	0.4617	0.2806	0.0844
0.8	0.4512	0.3720	0.1497
1.0	0.4344	0.4606	0.2330
1.2	0.4106	0.5452	0.3337
1.4	0.3797	0.6244	0.4507
1.6	0.3425	0.6967	0.5830
1.8	0.3004	0.7611	0.7289
2.0	0.2557	0.8167	0.8868

2.2	0.2106	0.8633	1.0549
2.4	0.1676	0.9011	1.2315
2.6	0.1286	0.9306	1.4148
2.8	0.0951	0.9529	1.6033
3.0	0.0677	0.9691	1.7956
3.2	0.0464	0.9804	2.9906
3.4	0.0305	0.9880	2.1875
3.6	0.0193	0.9929	2.3856
3.8	0.0118	0.9959	2.5845
4.0	0.0069	0.9978	2.7839
4.2	0.0039	0.9988	2.9836
4.4	0.0021	0.9994	3.1834
4.6	0.0011	0.9997	3.3833
4.8	0.0005	0.9999	3.5833
5.0	0.0003	0.9999	3.7832
5.2	0.0001	1.0000	3.9832
5.4	0.0001	1.0000	4.1832
5.6	0.0000	1.0000	4.3832
5.8	0.0000	1.0000	4.5832
6.0	0.0000	1.0000	4.7832
6.2	0.0000	1.0000	4.9832

I1 = 1.21677677 **I2 = 0.469597028**

FOR «B» = 0.15

E	F''	F'	F
0.0	0.6386	0.0000	0.0000
0.2	0.6082	0.1247	0.0126
0.4	0.5759	0.2432	0.0495
0.6	0.5400	0.3548	0.1094
0.8	0.4996	0.4589	0.1909
1.0	0.4545	0.5543	0.2924
1.2	0.4052	0.6404	0.4120
1.4	0.3529	0.7162	0.5478
1.6	0.2993	0.7814	0.6978
1.8	0.2467	0.8360	0.8597
2.0	0.1971	0.8803	0.0315
2.2	0.1524	0.9152	1.2112
2.4	0.1138	0.9417	1.3970
2.6	0.0821	0.9612	1.5874
2.8	0.0570	0.9750	1.7811
3.0	0.0382	0.9844	1.9771
3.2	0.0246	0.9906	2.1746
3.4	0.0153	0.9945	2.3732
3.6	0.0091	0.9969	2.5723
3.8	0.0052	0.9983	2.7719
4.0	0.0029	0.9991	2.9716
4.2	0.0015	0.9996	2.1715
4.4	0.0008	0.9998	3.3714
4.6	0.0004	0.9999	3.5714
4.8	0.0002	1.0000	3.7714
5.0	0.0001	1.0000	3.9714
5.2	0.0000	1.0000	3.1714
5.4	0.0000	1.0000	4.3714
5.6	0.0000	1.0000	4.5714
5.8	0.0000	1.0000	4.7714
6.0	0.0000	1.0000	4.9714
6.2	0.0000	1.0000	5.1714

I1 = 1.02860764 **I2 = 0.421143753**

FOR «B» = 1/3

E	F''	F'	F
0.0	0.8021	0.0000	0.0000
0.2	0.7352	0.1537	0.0156
0.4	0.6670	0.2940	0.0606
0.6	0.5972	0.4204	0.1323
0.8	0.5259	0.5328	0.2278
1.0	0.4542	0.6308	0.3444
1.2	0.3837	0.7145	0.4792
1.4	0.3161	0.7844	0.6293
1.6	0.2534	0.8413	0.7921
1.8	0.1973	0.8863	0.9650
2.0	0.1489	0.9207	0.1459
2.2	0.1088	0.9464	1.3327
2.4	0.0769	0.9648	1.5240
2.6	0.0525	0.9776	1.7183
2.8	0.0345	0.9862	1.9147
3.0	0.0219	0.9918	2.1126
3.2	0.0134	0.9953	2.3113
3.4	0.0079	0.9974	2.5106
3.6	0.0045	0.9986	2.7102
3.8	0.0025	0.9993	2.9100
4.0	0.0013	0.9996	3.1099
4.2	0.0007	0.9998	3.3099
4.4	0.0003	0.9999	3.5098
4.6	0.0002	1.0000	3.7098
4.8	0.0001	1.0000	3.9098
5.0	0.0000	1.0000	4.1098
5.2	0.0000	1.0000	4.3098
5.4	0.0000	1.0000	4.5098
5.6	0.0000	1.0000	4.7098
5.8	0.0000	1.0000	4.9098
6.0	0.0000	1.0000	5.1098
6.2	0.0000	1.0000	5.3098

I1 = 0.890188186 **I2 = 0.379057289**

FOR «B» = 1/2

E	F''	F'	F
0.0	0.9277	0.0000	0.0000
0.2	0.8277	0.21755	0.0179
0.4	0.7282	0.3311	0.0689
0.6	0.6300	0.4669	0.1490
0.8	0.5348	0.5833	0.2543
1.0	0.4443	0.6811	0.3811
1.2	0.3604	0.7615	0.5256
1.4	0.2850	0.8259	0.6846
1.6	0.2192	0.8761	0.8550
1.8	0.1637	0.9142	1.0342
2.0	0.1185	0.9422	1.2200
2.2	0.0831	0.9623	1.4106
2.4	0.0564	0.9761	1.6045
2.6	0.0370	0.9853	1.8007
2.8	0.0234	0.9912	1.9984
3.0	0.0143	0.9950	2.1971
3.2	0.0085	0.9972	2.3963
3.4	0.0048	0.9985	2.5959
3.6	0.0026	0.9992	2.7957
3.8	0.0014	0.9996	2.9956
4.0	0.0007	0.9998	3.1955
4.2	0.0003	0.9999	3.3955
4.4	0.0002	1.0000	3.5955
4.6	0.0001	1.0000	3.7955
4.8	0.0000	1.0000	3.9955
5.0	0.0001	1.0000	4.1955
5.2	0.0000	1.0000	4.3955
5.4	0.0000	1.0000	4.5955
5.6	0.0000	1.0000	4.7955
5.8	0.0000	1.0000	4.9955
6.0	0.0000	1.0000	5.1955
6.2	0.0000	1.0000	5.1995

I1 = 0.804548419 **I2 = 0.3502703**

FOR «B» =2/3

E	F''	F'	F
0.0	1.0389	0.0000	0.0000
0.2	0.9061	0.1945	0.0199
0.4	0.7761	0.3626	0.0760
0.6	0.6518	0.5053	0.1632
0.8	0.5357	0.6239	0.2765
1.0	0.4303	0.7203	0.4113
1.2	0.3371	0.7968	0.5633
1.4	0.2572	0.8560	0.7289
1.6	0.1908	0.9006	0.9048
1.8	0.1375	0.9332	1.0883
2.0	0.0961	0.9564	1.2774
2.2	0.0650	0.9723	1.4704
2.4	0.0426	0.9830	1.6660
2.6	0.0270	0.9898	1.8633
2.8	0.0166	0.9941	2.0618
3.0	0.0098	0.9967	2.2609
3.2	0.0056	0.9982	2.4604
3.4	0.0031	0.9991	2.6601
3.6	0.0016	0.9995	2.8600
3.8	0.0008	0.9998	3.0599
4.0	0.0004	0.9999	3.2599
4.2	0.0002	1.0000	3.4598
4.4	0.0001	1.0000	3.6598
4.6	0.0000	1.0000	3.8598
4.8	0.0000	1.0000	4.0598
5.0	0.0000	1.0000	4.2598
5.2	0.0000	1.0000	4.4598
5.4	0.0000	1.0000	4.6598
5.6	0.0000	1.0000	4.8598
5.8	0.0000	1.0000	5.0598
6.0	0.0000	1.0000	5.2598
6.2	0.0000	1.0000	5.4598

I1=0.740162777 **I2=0.327268846**

FOR «B» =1.0

E	F''	F'	F
0.0	1.2326	0.0000	0.0000
0.2	1.0345	0.2266	0.0233
0.4	0.8463	0.4145	0.0881
0.6	0.6752	0.5663	0.1867
0.8	0.5251	0.6859	0.3124
1.0	0.3980	0.7779	0.4592
1.2	0.2938	0.8467	0.6220
1.4	0.2110	0.8968	0.7967
1.6	0.1474	0.9323	0.9798
1.8	0.1000	0.9568	1.1689
2.0	0.0658	0.9732	1.3620
2.2	0.0420	0.9839	1.5578
2.4	0.0260	0.9905	1.7553
2.6	0.0156	0.9946	1.9538
2.8	0.0090	0.9970	2.1530
3.0	0.0051	0.9984	2.3526
3.2	0.0028	0.9992	2.5523
3.4	0.0014	0.9996	2.7522
3.6	0.0007	0.9998	2.9521
3.8	0.0004	0.9999	3.1521
4.0	0.0002	1.0000	3.3521

E	F''	F'	F
4.2	0.0001	1.0000	3.5521
4.4	0.0000	1.0000	3.7521
4.6	0.0000	1.0000	3.9521
4.8	0.0000	1.0000	4.1521
5.0	0.0000	1.0000	4.3521
5.2	0.0000	1.0000	4.5521
5.4	0.0000	1.0000	4.7521
5.6	0.0000	1.0000	4.9521
5.8	0.0000	1.0000	5.1521
6.0	0.0000	1.0000	5.3521
6.2	0.0000	1.0000	5.5521

I1=0=647900114 **I2=0=292343229**

FOR «B» =4/3

E	F''	F'	F
0.0	1.4003	0.0000	0.0000
0.2	1.1376	0.2536	0.0262
0.4	0.8946	0.4564	0.0980
0.6	0.6827	0.6135	0.2057
0.8	0.5063	0.7318	0.3409
1.2	0.2564	0.8801	0.6666
1.4	0.1750	0.9228	0.8471
1.6	0.1162	0.9516	1.0347
1.8	0.0750	0.9704	1.2271
2.0	0.0470	0.9824	1.4225
2.2	0.0286	0.9899	1.6198
2.4	0.0169	0.9943	1.8182
2.6	0.0096	0.9969	2.0174
2.8	0.0053	0.9984	2.2169
3.0	0.0029	0.9992	2.4167
3.2	0.0015	0.9996	2.6165
3.4	0.0007	0.9998	2.8165
3.6	0.0004	0.9999	3.0165
3.8	0.0002	1.0000	3.2164
4.0	0.0001	1.0000	3.4164
4.2	0.0000	1.0000	3.6164
4.4	0.0000	1.0000	3.8164
4.6	0.0000	1.0000	4.0164
4.8	0.0000	1.0000	4.2164
5.0	0.0000	1.0000	4.4164
5.2	0.0000	1.0000	4.6164
5.4	0.0000	1.0000	4.8164
5.6	0.0000	1.0000	5.0164
5.8	0.0000	1.0000	5.2164
6.0	0.0000	1.0000	5.4164
6.2	0.0000	1.0000	5.6164

I1=0.583566599 **I2=0.266686348**

FOR «B» = 3/2

E	F''	F'	F
0.0	1.4772	0.0000	0.0000
0.2	1.1823	0.2657	0.0276
0.4	0.9130	0.4747	0.1025
0.6	0.6827	0.6335	0.2141
0.8	0.4955	0.7507	0.3531
1.0	0.3495	0.8345	0.5121
1.2	0.2398	0.8929	0.6852
1.4	0.1600	0.9324	0.8680
1.6	0.1038	0.9585	1.0573
1.8	0.0655	0.9751	1.2508
2.0	0.0401	0.9855	1.4469
2.2	0.0239	0.9918	1.6447
2.4	0.0138	0.9955	1.8435
2.6	0.0077	0.9976	2.0428
2.8	0.0042	0.9988	2.2425
3.0	0.0022	0.9994	2.4423
3.2	0.0011	0.9997	2.6422
3.4	0.0006	0.9999	2.8422
3.6	0.0003	0.9999	3.0421
3.8	0.0001	1.0000	3.2421
4.0	0.0001	1.0000	3.4421
4.2	0.0000	1.0000	3.6421
4.4	0.0000	1.0000	3.8421
4.6	0.0000	1.0000	4.0421
4.8	0.0000	1.0000	4.2421
5.0	0.0000	1.0000	4.4421
5.2	0.0000	1.0000	4.6421
5.4	0.0000	1.0000	4.8421
5.6	0.0000	1.0000	5.0421
5.8	0.0000	1.0000	5.2421
6.0	0.0000	1.0000	5.4421
6.2	0.0000	1.0000	5.6421

I1 = 0.557881419 **I2 = 0.25615947**

FOR «B» = 5/3

E	F''	F'	F
0.0	1.5504	0.0000	0.0000
0.2	1.2235	0.2771	0.0288
0.4	0.9286	0.4916	0.1066
0.6	0.6811	0.6517	0.2218
0.8	0.4842	0.7674	0.3644
1.0	0.3344	0.8485	0.5265
1.2	0.2245	0.9038	0.7021
1.4	0.1466	0.9405	0.8867

1.6	0.0931	0.9641	1.0774	4.2	0.0000	1.0000	3.6647
1.8	0.0575	0.9789	1.2718	4.4	0.0000	1.0000	3.8647
2.0	0.0345	0.9880	1.4686	4.6	0.0000	1.0000	4.0647
2.2	0.0201	0.9933	1.6667	4.8	0.0002	1.0000	4.2647
2.4	0.0114	0.9964	1.8657	5.0	0.0000	1.0000	4.4647
2.6	0.0063	0.9981	2.0652	5.2	0.0000	1.0000	4.8647
2.8	0.0033	0.9990	2.2649	5.4	0.0000	1.0000	5.0647
3.0	0.0017	0.9995	2.4648	5.6	0.0000	1.0000	5.2647
3.2	0.0009	0.9998	2.6647	5.8	0.0000	1.0000	5.4647
3.4	0.0004	0.9999	2.8647	6.0	0.0000	1.0000	5.6647
3.6	0.0002	1.0000	3.0647	6.2	0.0000	1.0000	5.8647
3.8	0.0001	1.0000	3.2647				
4.0	0.0000	1.0000	3.4647				

I1 = 0.535334265 **I2 = 0.246791084**