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VELOCITY CONTROL IN TRAPEZOIDAL CHANNELS

by

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ABSTRACT

Linear proportional flow weirs commonly known as Sutro Weirs are extensively used for obtaining constant velocity in rectangular channels such as grit chambers of rectangular cross-section. In the present investigation, a weir is designed to achieve a constant velocity in a channel of trapezoidal cross-section. Experimental results show a good agreement with the theoretical analysis. This may find practical application in the design of grit chambers of trapezoidal shape or for dosing operations in industries where the velocity is to be controlled in a flow area of trapezoidal shape.

INTRODUCTION

Proportional weirs are a special type of weirs consisting normally a base weir and a proportional portion designed to achieve a certain desired velocity or discharge - head relation. The shape of base weir may be rectangular, triangular, trapezoidal, parabolic or any other shape (Fig.1). Linear proportional, logarithmic and quadratic weirs have been studies extensively with bottoms or base of different shapes [1,2]. An extensive bibliography on the subject was presented by Rao and Bhukari[3]. Theoretical Analysis consider a discharge of the form

$$Q = KH^n \qquad (1)$$

where

Q is the discharge over the weir

H is the head over the weir

n is an exponent

K is a constant

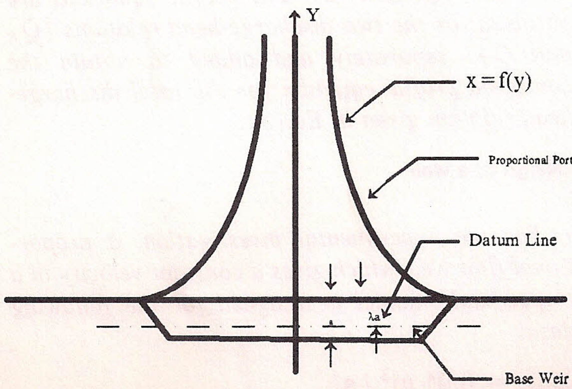
There arises a problem when n is less than 3/2, as the base width of such weir tends to infinity. This could easily be seen from the profile equation of the weir, x=f(y) established by Ricco [4] for a discharge relation of the form,

$$Q = C_d \sqrt{2g}\, a_1 H^n$$

$$x = f(y) = \frac{n(2n-1)\,\Gamma(n)}{\sqrt{\pi}\;\Gamma(n+1/2)}\, a_1 y^{n-3/2} \qquad (2)$$

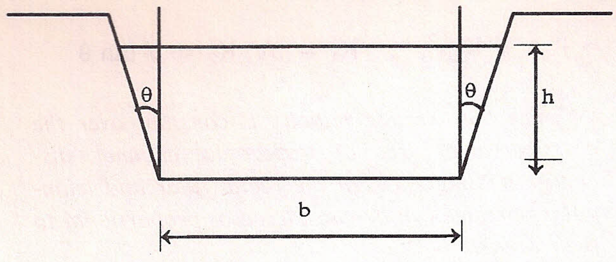
in which a_1, C_d are constants and g is the acceleration due to gravity .

In linear proportional flow weirs, the problem of the weir base tending to infinity is overcome by providing rectangular, triangular, trapezoidal or parabolic base at the bottom of the weir. In such cases, the datum above which the head has to be measured will not be at the weir crest, but at a certain height above the crest of the weir depending on the shape provided as the base. In the case of a rectangular base, for example, the datum is 1/3 the height of the rectangular portion above the crest. The aim of the present investigation is to design a weir for a channel of trapezoidal cross-section which gives a constant velocity of flow for various depths of flow.



(Fig.1) DEFINITION SKETCH.

Consider a channel with a. trapezoidal cross-section with base width b, depth of flow h and the sloping sides make an angle θ with the vertical (Fig.2).



(Fig. 2) CROSS-SECTION OF TRAPEZOIDAL CHANNEL-
Cross-sectional area of the channel,

$$A = V\,bh + h^2 \tan\theta \qquad (3)$$

Let V be the desired mean velocity of flow through the trapezoidal channel for all depths. Then,

$$Q = V\,bh + Vh^2 \tan\theta \qquad (4)$$

since V, b, θ, are constants, let $Vb = K_1$ and $V \tan\theta = K_2$ where K_1 and K_2 are constants. Hence

$$Q = K_1 h + K_2 h^2 \qquad (5)$$

or

$$Q_R = K_1 h, \quad Q_T = K_2 h^2 \quad \text{and} \quad Q = Q_R + Q_T$$

Q_R is the discharge through the rectangular portion and Q_T is the discharge through the triangular portion of the trapezoidal channel. The weir profile equation for any head-discharge relation can be obtained from the relation established by Ricco, equation (2). The profile equations are obtained for the two discharge-head relations (Q_R and Q_T) separately and added to obtain the combined profile equation for the total discharge-head relation given in Eq.(5).

Design of a weir

For the experimental investigation, a proportional flow weir which gives a constant velocity in a trapezoidal channel is designed for the following data:

$Q_{max.} = 0.01\ m^3 / s$,

width of channel, b = 8.75 cms.

and side slope of channel 7° with the vertical. These values are adopted to suit the laboratory conditions.

Let the average velocity of flow V = 0.5 m/s. Discharge through the trapezoidal channel,

$$Q = (bh + \tan\theta\, h^2)\,V$$

$$= K_1 h + K_2 h^2 \ , \quad K_1 = bV,\ K_2 = V\tan\theta$$

Since the average velocity is constant over the cross-sectional area of trapezoidal channel, discharge passing through the rectangular and triangular portions can be considered as proportional to their areas.

Thus if Q_R and Q_T are the discharges through the rectangular and triangular portions of the trapezoidal channel respectively,

$$Q_R = K_1 h \qquad \text{and} \qquad Q_T = K_2 h^2$$

Now, it is required to design the weir profile for discharge-head relations given above. Then, by combining the weir profiles obtained separately for above relations, we obtain the required weir profile for the trapezoidal channel, which yields constant velocity of flow at all depths. Let us consider the discharge-head relation for the rectangular portion,

$$Q_R = K_1 h$$

The weir profile for this relationship is a linear proportional weir as shown in Fig.3. Providing a rectangular base for the linear proportional weir, we have [2]

$$k_1 = \frac{2}{3} C_d\, b\, \sqrt{2g}\, a^{3/2} \frac{1}{\lambda\, a} \qquad (6)$$

$\lambda = \frac{2}{3}$ for rectangular base.

$$\text{Hence } K1 = C_d b \sqrt{2g}\, a \qquad (7)$$

$$\text{But } K1 = b.V = 0.0875 \times 0.5 = 0.04375$$

Assuming Cd = 0.65, a = 0.03 m. from Eq.(7)

The equation for the weir profile is given by the following equation,

$$2X_1 = b\left[1 - \frac{2}{\pi}\tan^{-1}\sqrt{\frac{y}{a}}\ \right] \qquad (8)$$

The values of X_1 is calculated for various values of y and are tabulated in Table 1.

*Now let us consider the **discharge-head relation** for the triangular portion, $Q_T = K_2 h^2$. The weir profile for this relationship can be **obtained from** Eq.(2) as follows:*

$$2X_2 = \frac{n(n-1)\,\Gamma(n)\ a_1\, y^{n-3/2}}{\sqrt{\pi}\ \Gamma(n+1/2)}$$

where n = 2

$$2X_2 = \frac{8}{\pi} a_1\, y^{1\backslash 2} \qquad (9)$$

$$\text{But } K_2 = V\tan\theta = C_d \sqrt{2g}\, a_1 \qquad (10)$$

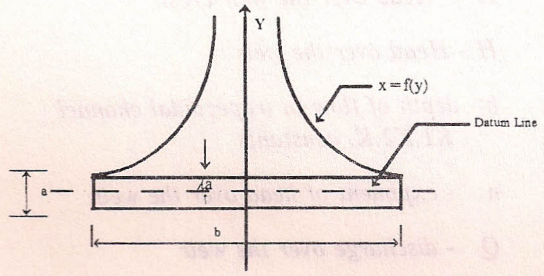
Hence $a_1 = 0.02132$ assuming $C_d = 0.65$

Now X_2 can be calculated for various values of y and the results are tabulated in Table 1. The required weir profile is obtained by adding X1 and X_2 for varous of y. The weir profile is shown in Fig. 4

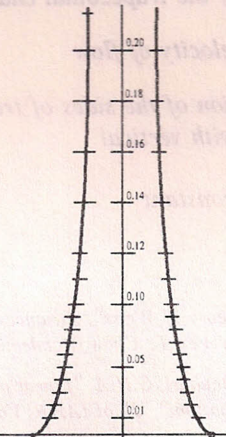
TABLE - 1 Co-ordinates of Weir Profile

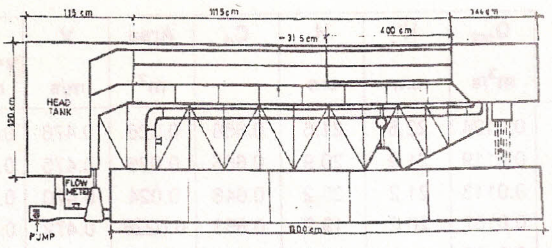
y (m)	X_1 (m)	X_2 (m)	$X=X_1+X_2$ (m)
-0.03	0.0438	-	0.0438
-0.02	0.0438	0.0	0.0438
-0.01	0.0438	0.0027	0.0465
0.00	0.0438	0.0039	0.0477
0.01	0.029	0.0047	0.0337
0.02	0.025	0.0055	0.0305
0.03	0.022	0.0061	0.0281
0.04	0.020	0.0067	0.0267
0.05	0.018	0.0072	0.0252
0.06	0.017	0.0077	0.0247
0.07	0.016	0.0086	0.0246
0.08	0.015	0.0095	0.0245
0.10	0.014	0.0102	0.0242
0.12	0.013	0.0109	0.0239
0.14	0.012	0.0116	0.0236
0.16	0.011	0.0122	0.0232
0.18	0.011	0.0128	0.0238
0.20	0.010	0.0134	0.0234
0.22	0.009	0.0139	0.0229

◆ *It should be noted that since the datum line for the linear proportional weir is at a height of 1/3a = 1 cm above the crest, the origin of X2 is at y = -0.02 m.*



(Fig.3) LINEAR PROPORTIONAL WEIR WITH RECTANGULAR BOTTOM.





(Fig. 4) WEIR PROFILE

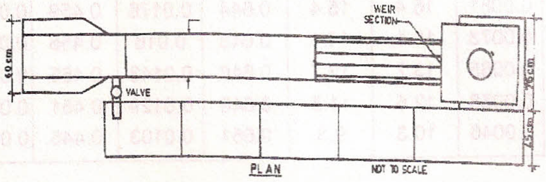


Fig 5 EXPERIMENTAL SETUP

Experimental Investigation.

The experimental set-up consists of a head tank, flume, test-weir, collecting tank, pump, flow meter and point gage for head measurement. The details of the set-up are shown in Fig.5. The trapezoidal horizontal flume 4m long, 8.75 cm. wide and 30 cm. high is provided at the end of the rectangular flume 7.0 m long, 30 cm wide and 30 cm high.

The test weir is prepared out of 3 mm. thick mild steel plate with a chamfer of 45 at the edge. The actual discharge through the weir is measured from the flow meter for various head over the weir section. The head over the weir section is measured using a point gage.

Analysis of results

The experimental results are tabulated in Table 2. Cd and V are calculated as follows:

$$Q_{act.} = C_d \left[b \sqrt{2g} \sqrt{a} \; H + \sqrt{2g} \; a_1 H^2 \right]$$

$$= C_d \left[0.0671\, H + 0.095\, H^2 \right] \quad (11)$$

Area of cross-section, $A = bh + \tan\theta \, . \, h^2$

$$= 0.0875\, h + 0.12278\, h^2 \quad (12)$$

$$V = Q_{act} \, ./A \quad (13)$$

The results; of C_d , V are tabulated in Table.2.

If V = 0.5 m/s (as assumed),

$K_1 = bV = 0.04375$ and $K_2 = V \tan\theta = 0.06139$.

Hence $Q = K_1 h + K_2 h^2$

$$= 0.04375\, h + 0.06139\, h^2 \quad (14)$$

This result is also tabulated in Table 2.

	Q_{act} m³/s	H' cm	H cm	C_d	Area m²	V m/s	q(if v=0.5/s) m³/s
1	0,0124	22.6	21.6	0.656	0.026	0.476	0.0130
2	0.0119	21.9	20.9	0.655	0.025	0.475	0.0125
3	0.0113	21.2	20.2	0.648	0.024	0.470	0.0120
4	0.0107	20.2	19.2	0.653	0.0226	0.472	0.0113
5	0.0102	19.7	18.7	0.643	0.022	0.464	0.0110
6	0.0094	18.4	17.4	0.646	0.020	0.464	0.0101
7	0.0087	17.4	16.4	0.642	0.189	0.459	0.00947
8	0.0081	16.4	15.4	0.644	0.0176	0.459	0.00883
9	0.0073	15.1	14.1	0.643	0.016	0.456	0.00801
10	0.0065	13.7	12.7	0.646	0.0143	0.455	0.00715
11	0.0058	12.5	11.5	0.646	0.0129	0.451	0.00643
12	0.0046	10.3	9.3	0.651	0.0103	0.446	0.00516

Table 2 (EXPERIMENTAL RESULTS).

H' - Head above the crest, in this case H' = h.

H - Head above the datum of weir.

Q_{act} *and Q are plotted against H in Fig.6.*

Discussion and Conclusion.

It could be seen from the results that Cd is nearly constant for various depths of flow. The average velocity is also nearly constant for various depths of flow, but the value is slightly lower than the required value of 0.5 m/s. This is due to the fact that there is differences of 1 cm. between the depth of flow in the trapezoidal channel and the head over the weir. If the two depths a4re the same, it could be seen from Fig. 6 that the experimental Q and the Q calculated on the basis of the average velocity equals to 0.5 m/s are the same for the same depth. More experimental results are necessary to drawn a more definite conclusion. However, it can be concluded that the experimental results justify the validity of the method of approach to the problem.

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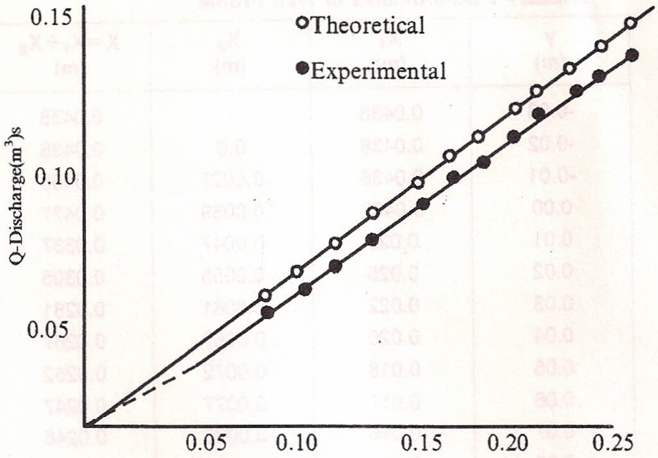

(Fig...6) RELATION BETWEEN DISCHARGE & HEAD OVER THE WEIR.

Notations

A - Cross-sectional area of trapezoidal channel

a - depth of crest below y-axis a1 - constant

b - base width

Cd - coefficient of discharge

g - acceleration due to gravity

H' - Head over the weir crest

H - Head over the weir

h - depth of flow in trapezoidal channel K1,K2,K, constants

n - exponent of head over the weir

Q - discharge over the weir

QR - discharge through the rectangular portion of trapezoidal channel

QT - discharge through the triangular portion of the trapezoidal challe

V - mean velocity of flow

e - inclination of the sides of trapezoidal channel with vertical

λ - datum constant

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