PREDICTION OF MIXED CONVECTION IN INCLINED SEMICIRCULAR DUCTS

A. A. Busedra

Mechanical Engineering Department,
University of Garyounis, Benghazi, Libya
Email: kbusedra@garyounis.edu

الملخص

تم فى هذا البحث دراسة الحمل المختلط للانسياب الرقائقى في أنابيب نصف دائرية ومائلة بزاوية إلى أعلى (سطحها المستوى في الاتجاه العمودي) وذلك بتطبيق حرارة منتظمة محورياً وفيض حراري منتظم محيطياً (H2) تَمت دراسة هذه الأنابيب في حالتي الوضع الأفقي والوضع المائل بزاوية 30 درجة عن طريق تحليل المعادلات الحاكمة للسرعات ودرجات الحرارة تحليلها عددياً باستعمال طريقة (Finite–control volume approach). فيما يتعلق بنتائج إنتقال الحرارة فقد تم الحصول عليها للحالتين بإدخال متغيرات مثل زوايا الميول (0^o، 30^o) و$Pr = 7$ و $Re = 500$ وقيم عديدة لرقم *Grashof*. أوضحت نتائج توزيع السرعات ودرجات الحرارة وحرارة الجدار الأنبوب ورقم *Nusselt* وأن تأثير الزاوية ($\alpha = 30^o$) مهم لأنبوب نصف دائري الشكل نظراً لاختفاء الطبقات الحرارية عند ميول الأنبوب إلى أعلى وخاصة عند أعلى قيمة لرقم *Grashof* مما نتج عنه زيادة كبيرة في رقم *Nusselt*. أما بالنسبة لوضع الأنبوب أفقياً فإن زيادة رقم *Grashof* ليس له تأثير ملحوظ على إختفاء الطبقات الحرارية نتيجة للتغير القوى في درجات حرارة حول محيط الأنبوب.

ABSTRACT

Laminar mixed convection in semicircular ducts inclined upward (with the flat wall in a vertical position) is investigated for the case of uniform heat input axially with uniform wall heat flux circumferentially, H2. Two inclinations are considered in this study, the horizontal orientation and 30^o inclination. The governing equations for the velocity and temperature are solved numerically by using a finite control volume approach. Results of the heat transfer characteristics are obtained for the two cases using input dimensionless parameters, such as the inclination angle (0^o and 30^o), $Pr = 7$, $Re = 500$, and a wide range of Grashof number. These computed results include the velocity and temperature distributions, wall temperature, and Nusselt number. The influence of inclination ($\alpha = 30^o$) is found to be important in semicircular ducts due to the disappearance of the thermal stratification with this upward inclination, particularly at high Grashof number. As a result, a significant increase in Nusult number is noticed. For the horizontal case, however, increasing Grashof number has no observable effect on the disappearance of the thermal stratification due to the strong variation in the wall temperature around the circumference.

KEYWORDS: Inclined semicircular ducts; Laminar; Mixed convection; Heat flux

INTRODUCTION

The need of noncircular passages used in compact heat exchangers has motivated extensive research into various applications of flow and heat transfer augmentation. Therefore, attention has been devoted in recent years to the investigation of combined free and forced laminar convection in inclined ducts of various cross sections. Iqbal and Stchiewicz [1] studied the problem of laminar fully developed flow in inclined circular tubes, using the perturbation power-series solution, with uniform heat flux at the wall. They reported that, *Nu* reaches a maximum at some inclination angle between the horizontal and vertical inclinations. Cheng and Hong [2] studied the same problem considered in Ref [1] using the boundary vorticity method and reported a substantial difference between their *Nu* and those in Ref [1]. Orfi and Galanis [3] solved also the same problem using the finite control volume approach. They found that for a given fluid and Grashof number an optimum tube inclination exists, which maximizes *Nu*. Other results for laminar, fully developed mixed convection were reported for inclined circular tubes [4], for rectengulat ducts [5] and for inclined parallel plated [6].

Busedra and Soliman [7] studied the problem of laminar, fully developed mixed convection in inclined semicircular ducts (the flat wall in the vertical position) under buoyancy assisted and buoyancy opposed conditions. Two thermal boundary conditions were used; H1 (uniform heat input axially with uniform wall temperature circumferentially).and H2 The governing equations were solved numerically by using the finite control volume approach They reported that, for both H1 and H2 conditions with high *Gr,* Nusselt number develops a trend whereby their value increases with α up to a maximum and then decreases with further increase in α. Busedra and Soliman [8] studied the combined free and forced convection experimentally for laminar water flow in the entrance region of a semicircular duct (the flat surface in a vertical position), with upward and downward inclinations within $\pm 20^o$. They concluded that for the upward inclinations, *Nu* increases with *Gr* and with α up to 20^o. Other previous studies that dealt with mixed convection for the H1 thermal boundary condition case in horizontal semicircular ducts were reported by Nandakumar et.al. [9], their investigation was for laminar fully developed mixed convection in a horizontal semicircular duct with the flat wall at the bottom. Lei and Trupp [10] solved the same problem considered in Ref [9] with the flat wall on top. They reported approximately the same results of *Nu* as for the flat wall at the bottom (Ref [9]**)**. Chinporoncharoenpong et.al. [11] Studied also the same problem by rotating the semicircular duct from 0^o (the flat wall on top) to 180^o (the flat wall at the bottom) with an incremental angle of 45^o. They found that, orienting the flat wall of the semicircular duct vertically 90^o up to 135^o gave the highest *Nu* among the other orientations. Busedra and El-Abeedy [12] studied recently the same problem considered in Ref [11] by inclining the semicircular duct at a fixed angle of α = 20^o. They noted that, the effect of duct inclination on the orientation is important in heat transfer enhancement, particularly when orienting the flat wall of the duct vertically.

Since no analysis is available for inclined semicircular ducts with the H2 thermal boundary condition, other than Ref [7], the objective of the present investigation is to generate theoretical results for fully developed laminar mixed convection in heated semicircular ducts with horizontal and upward inclinations. The parameters, such as the inclination angle (0^o and 30^o), *Pr* = 7, *Re* = 500, and a wide range of Grashof number were considered, to see their effects on the velocity and temperature distributions, wall temperature, and Nusselt number.

MATHEMATICAL FORMULATION

Figure (1) shows the geometry of an inclined semicircular duct with the flat wall always falling in a vertical position. The fluid is incompressible, steady laminar and fully developed hyrodynamically and thermally. Viscous dissipation is assumed to be negligible. Fluid properties are assumed to be constant, except for the density variation in the buoyancy terms, which varies with temperature according to the Boussinesq approximation. Heat input is assumed to be uniform axially with uniform wall heat flux circumferentially.

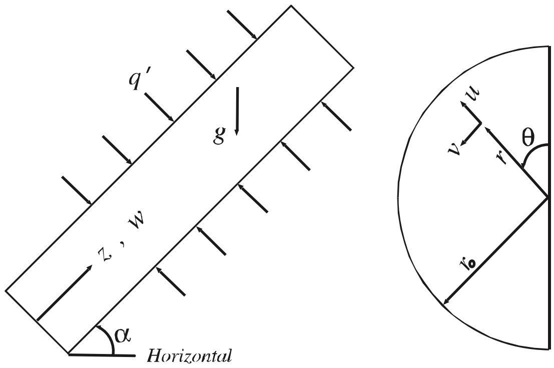

Figure1: Geometry and coordinate system.

The fluid pressure decomposition which quite widely used is given by Ref [13].

$$p(r,\theta,z) = p_1(z) + p_2(r,\theta) \tag{1}$$

where p_1 is the cross-sectional average pressure, which is assumed to vary linearly in the axial direction, while p_2 is the pressure variation around the duct cross-section. The dimensionless parameters and variables are also introduced as follows:

$$\begin{aligned}
&R=\frac{r}{r_o},\quad Z=\frac{z}{r_o},\quad U=\frac{ur_o}{\nu},\quad V=\frac{vr_o}{\nu},\quad W=\frac{w}{w_m}\\
&T=\frac{t-\bar{t}_w}{q'/k},\quad P_1=\frac{p_1^* r_o}{\rho\nu w_m},\quad p_1^*=p_1+\rho_w gz\sin\alpha\\
&D_h=(\frac{2\pi}{\pi+2})r_o,\quad P_2=\frac{p_2^* r_o^2}{\rho\nu^2},\quad p_2^*=p_2+\rho_w gr\cos\alpha\cos\theta\\
&\mathrm{Re}=\frac{D_h w_m}{\nu},\quad \mathrm{Pr}=\frac{\rho\nu C_p}{k},\quad Gr=\frac{g\beta q' r_o^3}{k\nu^2}
\end{aligned} \tag{2}$$

With these definitions, the non-dimensional governing equations in cylindrical coordinates are:

Continuity equation:

$$\frac{1}{R}\frac{\partial(RU)}{\partial R}+\frac{1}{R}\frac{\partial(V)}{\partial\theta}=0 \tag{3}$$

Momentum equation:

R-direction:

$$U\frac{\partial U}{\partial R}+\frac{V}{R}\frac{\partial U}{\partial \theta}=-\frac{\partial P_2}{\partial R}+\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial U}{\partial R}\right)+\frac{1}{R^2}\frac{\partial^2 U}{\partial \theta^2}-\frac{2}{R^2}\frac{\partial V}{\partial \theta}-\frac{U}{R^2}+\frac{V^2}{R}+GrT\cos\alpha\cos\theta \tag{4}$$

θ-direction:

$$\left(U\frac{\partial V}{\partial R}+\frac{V}{R}\frac{\partial V}{\partial \theta}\right)=-\frac{1}{R}\frac{\partial P_2}{\partial \theta}+\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial V}{\partial R}\right)+\frac{1}{R^2}\frac{\partial^2 V}{\partial \theta^2}+\frac{2}{R^2}\frac{\partial U}{\partial \theta}-\frac{V}{R^2}-\frac{UV}{R}-GrT\cos\alpha\sin\theta \tag{5}$$

Z-direction:

$$U\frac{\partial W}{\partial R}+\frac{V}{R}\frac{\partial W}{\partial \theta}=-\frac{\partial P_1}{\partial Z}+\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial W}{\partial R}\right)+\frac{1}{R^2}\frac{\partial^2 W}{\partial \theta^2}+2\left(\frac{\pi}{\pi+2}\right)\left(\frac{Gr}{\mathrm{Re}}\right)T\sin\alpha \tag{6}$$

Energy Equation:

$$\Pr\left(U\frac{\partial T}{\partial R}+\frac{V}{R}\frac{\partial T}{\partial \theta}\right)=\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial T}{\partial R}\right)+\frac{1}{R^2}\frac{\partial^2 T}{\partial \theta^2}-\left(\frac{2}{\pi}\right)W \tag{7}$$

The applicable boundary conditions are:

$$\frac{\partial T}{\partial R}=\frac{1}{\pi+2}\ \text{at } R=1,\qquad \frac{\partial T}{\partial \theta}=-\frac{R}{\pi+2}\ \text{at } \theta=0,\qquad \frac{\partial T}{\partial \theta}=\frac{R}{\pi+2}\ \text{at } \theta=\pi \tag{8}$$

Another important parameter used in engineering design is Nusselt number, Nu, given by:

$$Nu=\frac{hD_h}{k}=-\frac{2\pi}{(\pi+2)^2}\frac{1}{T_b} \tag{9}$$

COMPUTATIONAL PROCEDURE

Governing equations, (Eqs (3)-(7)), were solved numerically using finite control volume approach Ref [14]. The dimensionless partial differential equations were discretized and the power law scheme was used for the treatment of the convection and diffusion terms. The velocity-pressure coupling was handled using the SIMPLER algorithm. A staggered grid was used in the computations with uniform subdivisions in the R and θ directions. The control volumes adjacent to the boundary were subdivided into two control volumes in order to capture the steep gradients in the temperature and axial velocity. For given values of the input parameters *Re, Pr, α,* and *Gr*, computations started with guessed field of *U, V, W,* and *T*. Typically the initial guess used was $U = V = W = T = 0$ at all mesh points. The discretized equations were solved simultaneously for each radial line using the tridiagonal matrix algorithm [TDMA], and the domain was covered by sweeping line by line in the angular direction. At the end of each iteration, a correction procedure was applied to the value of W, using the conservation of mass in order to insure that the mean value of the dimensionless axial velocity, W_m is equal to 1. As well, the average wall temperature was calculated and this value was subtracted from the temperature at all nodes, thus insuring an average wall temperature of zero.

Iterations continued until the three velocity components and the temperature at all grid points satisfied the following convergence criterion:

$$\left|\frac{\phi_{new}-\phi_{old}}{\phi_{new}}\right|\le 10^{-6} \tag{10}$$

Three different grid sizes for pure forced convection were used and the results are presented in Table (1). Comparing the results in Table (1) with the exact solutions of Nu_o = 2.923 in Ref [15], it can be seen that the 30 x 48 (R x θ) grid is sufficient to obtain accurate *Nu* value that is within 0.1% from the exact value. In view of computation time, this selected grid is a reasonable compromise between accuracy and computer time.

Table1: Effect of grid size on Nuo for Gr = 0.

Mish size	Nu_o
15 x 24	2.949
30 x 48	2.926
60 x 90	2.922
Exact value [15]	2.923

RESULTS AND DISCUSSION

Solutions were obtained for horizontal and inclined semicircular duct using the H2 thermal boundary condition. A wide range of Grashof was covered providing results for *Pr* = 7 and *Re* = 500 and α = 0^o and 30^o. The numerical solution was obtained for a representative sample of the velocity and temperature distributions, wall temperature, and the overall quantities of Nusselt number.

The velocity and temperature results are presented in Figures (2) and (3). For the horizontal orientation, Figure (2), the maximum velocity and minimum temperature shift to the lower part of the duct cross-section due to the secondary flow motion associated with free convection. Further, a strong variation in the wall temperature around the circumference causes temperature stratification with layers of hot fluid occupying the upper part of cross-section. The case of upward inclination illustrated in Figure (3). The maximum velocity shifts considerably towards the upper part of the cross-section and temperature stratification disappears indicating much less circumferential variation in the wall temperature.

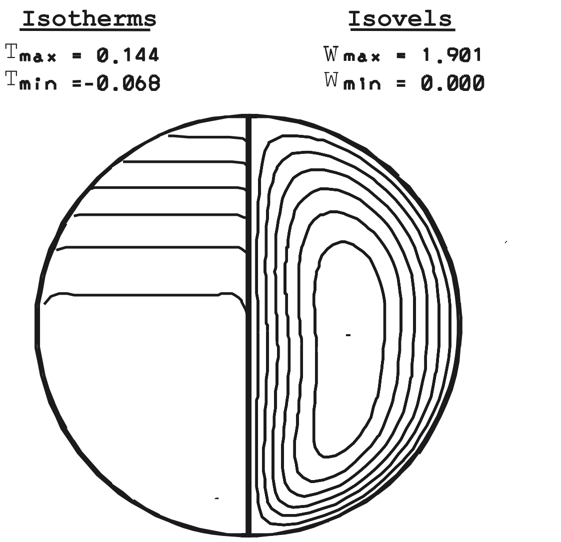

Figure 2: Velocity and temperature contours for α = 0^o with Gr = 2 x 10^6.

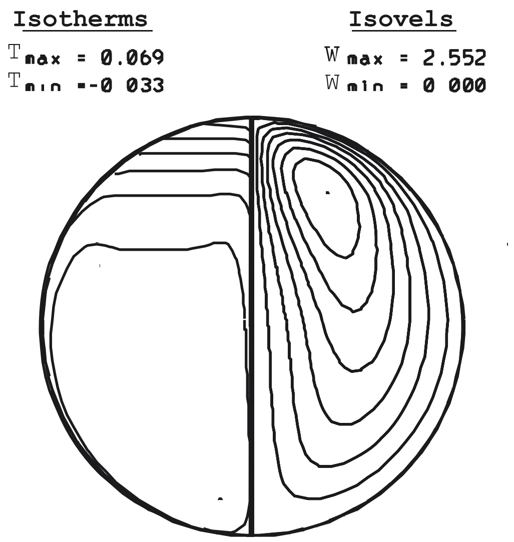

Figure 3: Velocity and temperature contours for α = 30o with Gr = 2 x 10^6.

The circumferential variation of wall temperature is presented in Figures (4) and (5). The forced convection case calculated at *Gr* = 0 is presented as a reference in all figures in order to observe the effect of free convection. For $\alpha = 0^o$, Figure (4), the wall temperature varies considerably around the circumference with high temperature in the upper part and low temperature in the lower part of the semicircular duct. The difference between the maximum (at the upper part) and the minimum (at the lower part) wall temperatures remains nearly constant for the three values of *Gr*.

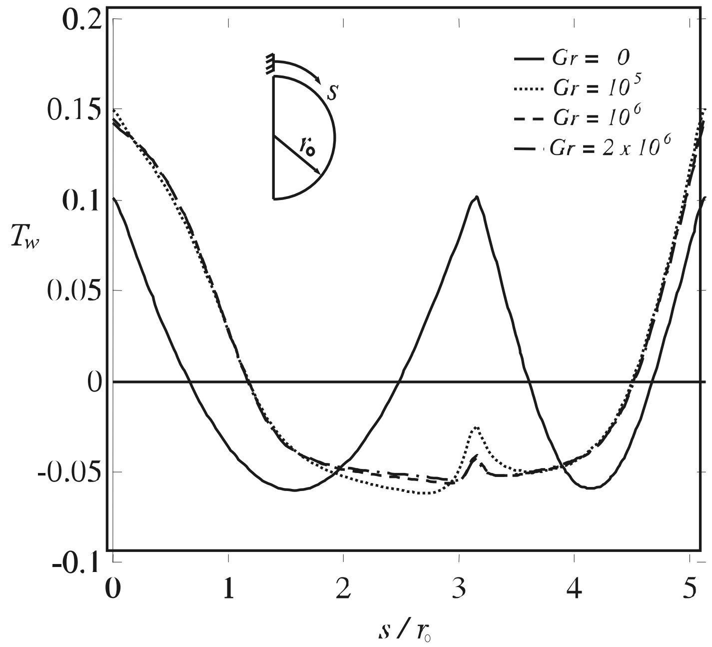

Figure 4: Circumferential variation of wall temperature for $\alpha = 0^o$.

As a result, increasing *Gr* has no observable effect on the disappearance of the thermal stratification due to the strong variation in the wall temperature around the circumference. This is consistent with the temperature distribution shown earlier in figure 2 for $\alpha = 0^o$, where the isotherms show temperature stratification for the value of *Gr*. For $\alpha = 30^o$, however, the difference between high and low wall temperatures is affected by the value of *Gr*. As can be seen (Figure 5), the difference between these extreme wall temperatures decreases as *Gr* increases. Therefore, the uniformity appearance of T_w may lead to increased heat transfer. This trend is consistent with the gradual disappearance of thermal stratification with increasing *Gr* for $\alpha = 30^o$, as noted earlier in Figure (3).

Figure (6) shows the behavior of Nusselt number as function of Grashof number. The general trend in these results is that, Nu/Nu_o increases with increasing *Gr*. The magnitude of this increase becomes large for $\alpha = 30^o$, due to the disappearance of the temperature stratification indicating much less circumferential variation in the wall temperature.

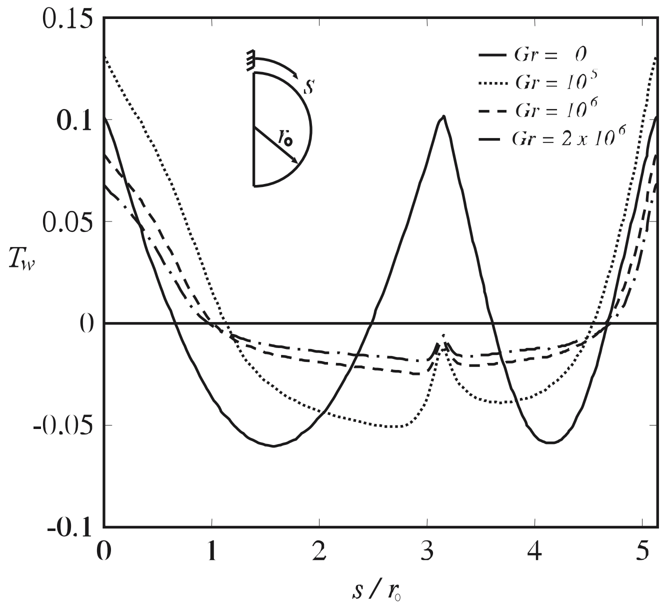

Figure 5: Circumferential variation of wall temperature for α = 30°.

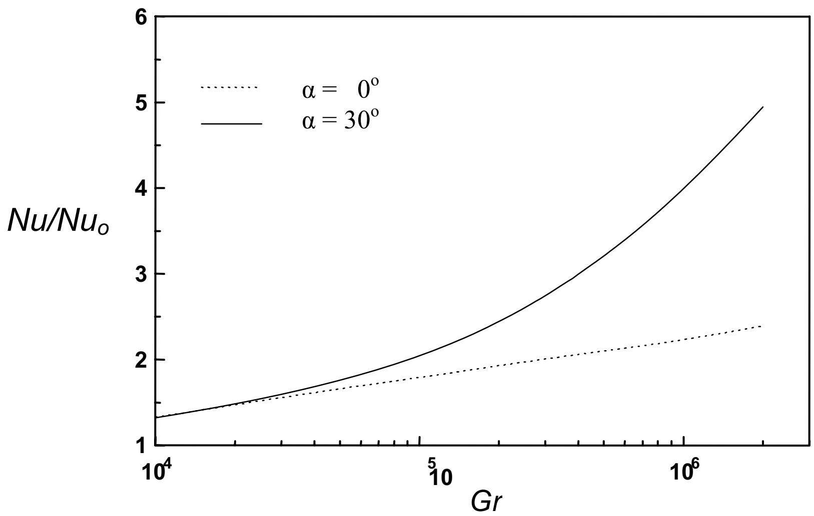

Figure 6: Nusselt number results with Gr.

CONCLUSIONS

The analysis has been utilized for laminar fully developed mixed convection in semicircular ducts inclined at 30° with the H2 thermal boundary condition. Results showed the following conclusions:

1. The maximum axial velocity, for the horizontal case, is found to be shifted to the lower part of the duct cross-section, while, for α = 30° the axial velocity shifted considerably towards the upper part of the cross-section.
2. For α = 0°, a strong variation in the wall temperature around the circumference caused temperature stratification with layers of hot fluid occupying the upper part of cross-section, while, for α = 30° the temperature stratification disappeared indicating much less circumferential variation in the wall temperature.

3. Increasing *Gr*, for $\alpha = 0^o$, has no observable effect on the disappearance of the thermal stratification. For $\alpha = 30^o$, however, the difference between high and low wall temperatures decreases as *Gr* increases.
4. A significant increase in Nusult number is found for the inclined case ($\alpha = 30^o$) as compared with the horizontal case.

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NOMENCLATURE

D_h	hydraulic diameter, m
g	gravitational acceleration, m/s^2
Gr	Grashof number
h	average heat transfer coefficient, W/m^2 K
H1, H2	thermal boundary conditions
k	thermal conductivity, W/mK
Nu	Nusselt number
P	pressure, N/m^2
p_1	cross-sectional average pressure, N/m^2
p_2	cross-sectional excess pressure, N/m^2
P_1	dimensionless cross sectional average pressure
P_1	dimensionless cross sectional excess pressure
P_r	Prandtl number
q'	rate of heat input per unit length, W/m
r	radial coordinate, m
r_o	radius of circular wall, m
R	dimensionless radial coordinate
t	temperature, K
T	dimensionless temperature
u, v, w	radial, angular, and axial velocities, m/s
U, V, W	dimensionless radial, angular, and axial velocities
z	axial coordinate, m
Z	dimensionless axial coordinate

Greek Letters

α	inclination angle
β	coefficient of thermal expansion, K^{-1}
ϕ	scalar function
θ	angular coordinate
ν	kinematic viscosity, m^2/s
ρ	density, kg/m^3

Superscripts

b	bulk value
m	mean value
w	at the wall
o	corresponding to Gr = 0