AXIAL COMPRESSION OF THIN-WALLED METAL TUBES

Ramadan O. Saied and F. M. Shuaeib

Mechanical Engineering Department,
Faculty of Engineering, Garyounis University
E-mail: saied972004@yahoo.com

الملخص

تقدم هذه الدراسة تحليلاً نظرياً وعملياً لتحطم الأنابيب المعدنية رقيقة الجدار وذات المقطع الدائري تحت الأحمال المحورية الساكنة وذلك لمعرفة مقدرتها على أمتصاص الطاقة الناتجة من عملية التحطيم و تحديد أساليب التحطم والأحمال المسلطة. ولغرض إجراء التجارب العملية تم إعداد العديد من عينات الأنابيب المعدنية المصنوعة من الصفائح المجلفنة ذات سمك قدره 1.5 مليمتر وبأقطار 100 و 150 و 200 مليمتر وأطوال مختلفة تتناسب مع هذه الأقطار. أدت نتائج التجارب التي أجريت إلى الحصول على مخطط الحمل مع الإزاحة لكل عينة متحطمة كما حددت أساليب التحطم بأربعة أنواع. من خلال هذة الدراسة تبين أن الطاقة الناتجة من تحطم الأنبوب تمتص من خلال المفاصل الثابتة والمتحركة المتكونة في جدار الأنبوب أثناء التحطم. كذلك وجد أن الحمل المتوسط والحمل الأقصى والطاقة الممتصة ونسبة الإجهاد ($\bar{\sigma}/\sigma_y$) تتناسب طردياً مع نسبة السمك الى القطر (T/D).

ABSTRACT

The present paper investigates the compression process of thin-walled metal tubes under axial static compression load. Several Specimens of tubes with different diameters 100 , 150 and 200 mm and lengths were manufactured using 1.5 thickness low carbon steel sheets (Zinc coated), in order to carry out axial compressive tests. Analytical in-extensional collapse mode was used to predict the mean load carrying capacity, energy absorption and stress ratios. Load-displacement curves were obtained experimentally and four types of collapsed modes were observed. It was noted that the energy of the crushing was absorbed by fixed, included and traveled plastic hinges. Mean load carrying capacity, energy absorption and stress ratio ($\bar{\sigma}/\sigma_y$) were found to be increase with increase of (t/D) ratio.

KEYWORDS: Crushing; Energy absorption; Tubular structure; Thin-walled tubes; Plastic hinges; In-extensional collapse mode; Mean load carrying capacity; Stress ratio.

INTRODUCTION

Tubular structures are among energy absorbing devices which can be used in many applications such as cars, trains, ships, airplanes and etc. Over recent years there has been a sustained interest in axial compression behavior of thin walled metal tubular structures. Early investigations [1-2] were concerned with rectangular cross-section metal tubes under dynamic and static loadings in order to understand the behavior of rail coach and vehicles body shells. Empirical relations for specific energy absorbing capacity and mean load carrying capacity were developed. These investigations have been extended to the circular cross-section metal tubes because of its simple geometry, low cost and high energy absorbing capability. Alexander [3] presented approximate analysis of the collapse mode and expression of mean load carrying capacity for circular tubes under axial loading. Johnson et al [4] developed an in-extensional collapse mode for the axial compression of thin tubes and predicted simple expressions for mean collapse load and the energy absorbed. Good agreement between experimental and predicted loads was observed. Johnson and Reid [5] reviewed several energy absorbing devices and they suggested that progressively crushing tubes under axial load are among the more efficient ones.

Wierzbicki [6] investigated basic, kinematical, admissible, global deformation mechanisms of thin walled tubes. Fold mechanism, in which large portions of the tube undergo in-extensional and extensional deformation has been described. These mechanisms or models have been applied to the crushing of both square and rectangular tubes. It has been demonstrated that plastic deformations caused by local stretching at edge of the fold, bending of the horizontal fixed hinges and bending of inclined hinges each dissipate equal amount of energy. Expressions for mean loads, fold length and fold radius were derived by equating the external work needed to completely collapse one fold to the plastic work done in deformation regions. Marzo et al [8] investigated the collapse and steady flow of thick-walled flexible tubes in three-dimension using finite-element approach. Their model results were in excellent agreement with published numerical values based on thin-shell elements. More recently Hamouda et al [9] studied the energy absorption capacities of three types of square tubular structures named as mild steel, glass reinforcement composite and Kevlar reinforcement composites and they found that parameters such as thickness, type of fibre and length of tubes have a considerable effect on the crushing behaviour of collapsed tubes.

EXPERIMENTS PROGRAM

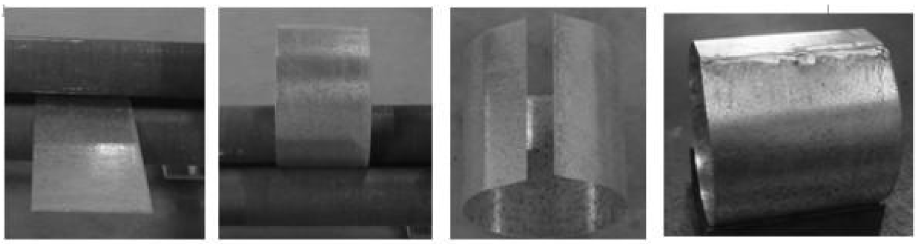
Preparation of specimens

ASTM A 570 GR A, low carbon steel sheets (zinc coated) of 1.5 mm thickness were used to fabricate twelve tubes of 100, 150 and 200 mm diameters. The yield stress and other mechanical specifications of the steel sheet were determined experimentally using tensile tests on coupons prepared from the same stock of sheet according to the S-A370 method [10] as shown in Table (1).

Table 1: Measured Specification of steel sheets used to manufacture the specimens.

Thickness (mm)	Yield strength (MN/m^2)	Tensile strength (MN/m^2)	Elongation %	Young's modulus (GN/m^2)
1.5	228	414	20	200

Four tubes of each diameter group were fabricated according to 1, 2, 3 and 4 (L/D) ratios. Each tube was manufactured by passing the sheet between two rollers of rolling machine several times to form a tubular shape. The two edges of the curved sheet were joined by arc welding along its axis see Figure (1). All the tubes were cleaned and painted with rust paint see Figure (2).



(a) (b) (c) (d)

Figure 1: Manufacturing process of the tube

(a) (b)

Figure 2: The specimens with different lengths and diameters, (a) before and (b) after painting

Quasi-static axial compression test

In order to determine the maximum collapsed load, deformation mode and load-displacement curves of the collapsed tubes, Quasi-static axial compression tests were performed on the tubes using Pivoal Universal Testing Machine at cross-head of 10 mm/min in the Material Laboratory of the Faculty of Engineering at Garyounis University.

RESULTS AND DISCUSSION

Tables (2a - 2c) summarise the experimental and theoretical results of the collapsed tubes. It can be noted that the maximum collapsed load, mean load carrying capacity, energy absorption and number of lobes were found to be increased slightly with increase of the L/D ratio. Whereas the stress ratio ($\bar{\sigma}/\sigma_y$), which was calculated by equations (14) and (16) is almost constant.

Table 2a: the experimental results of the collapsed tube of 100 mm diameter

L/D ratio	C' %	P_{max} (Expe) (kN)	P_{mean} (Exp) (kN)	P_{mean} (Theo) (kN)	W (Expe.) (J)	$\bar{\sigma}/\sigma_y$ (Theo.)	Collapse mode	r mm	S	n	m
1	80	121	53	58	2845	0.151	in-extensional	4	-	3	3
2	80	122	61	64	2768	0.151	in-extensional	4	-	3	6
3	80	130	63.5	--	2421	0.266	mixed	4	2	4	10
4	85	130	65.2	--	7047	0.151	mixed	5	2	3	10

Table 2b: the experimental results of the collapsed tube of 150 mm diameter

L/D ratio	C' %	P_{max} (Expe) (kN)	P_{mean} (Exp) (kN)	P_{mean} (Theo) (kN)	W (Expe.) (J)	$\bar{\sigma}/\sigma_y$ (Theo.)	Collapse mode	r mm	S	n	m
1	80	173	83	81	8021	0.151	in-extensional	4	-	4	5
2	76	181	85	83	3763	o.151	in-extensional	4	-	4	12
3	60	185	86	--	5130	0.250	mixed mode	4	-	4	12
4	55	185	86	88	7673	0.155	in-extensional	4	-	4	8

Table 2c: the experimental results of the collapsed tubes of 200 mm diameter

L/D ratio	C' %	P_{max} (Expe) (kN)	P_{mean} (Exp) (kN)	P_{mean} (Theo) (kN)	W (Expe.) (J)	$\bar{\sigma}/\sigma_y$ (Theo.)	Collapse mode	r mm	S	n	m
1	55	213	88	83	2253	0.133	In-extensional	4	-	4	8
2	57	215	88.5	79.9	4961	0.133	In-extensional	4	-	4	8
3	50	215	88.9	91	5354	0.133	In-extensional	4	-	4	8
4	50	217	93	91	5938	0.133	In-extensional	4	-	4	8

Deformation process

Figures (3 - 4) show photographs of the deformation process of the collapsed tube of 100 mm and 150 mm diameters with L/D ratio = 2 respectively. In Figure (3), the tube started to collapse from the bottom end in extensible manner to form two symmetric plastic rings as shown in the photograph (c). After that the tube buckled and collapsed due to instability in in-extensible manner developing plastic hinges in circumferential direction to form the first axial lobe followed by further lobes until the tube deformed completely as shown in the photograph (e). Whereas, the tube in Figure (4) started to collapse from top end and then deformed by in-extensible manner to form axial lobes until the tube deformed completely.

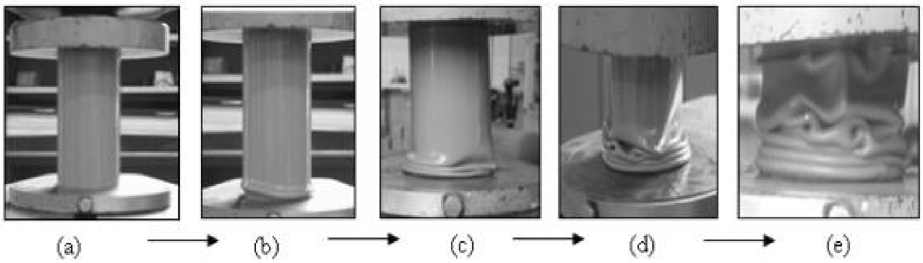


Figure 3: Deformation process of mixed mode of collapse for a 100 mm dia. tube with (L/D) = 2.

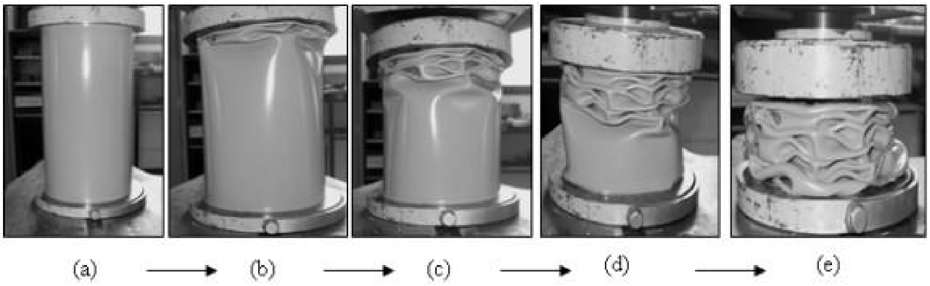


Figure 4: Deformation process of in-extensional mode of collapse for a 150 mm diameter tube with (L/D) = 2.

Load-displacement curves.

A load-displacement curve was obtained for each specimen during the compression test using the plotter of the testing machine. Figure (5) shows a typical load- displacement curve of collapsed tube (as described in Figure 3). The letters are corresponding to the deformation stages of the tubes. It was observed that all the curves of the collapsed tubes exhibited similar behaviour. The curve shows a large initial peak load corresponding to the initial collapse followed by rapid decrease in load due to the buckling of the tube. After this stage, the curve exhibited a series of fluctuations about the mean load. The fluctuations consist of peaks and troughs corresponding to deformation and folding of the tube. The main use of the load-displacement curve is to estimate maximum load carrying capacity, mean load carrying capacity and to evaluate the energy absorption by calculating the area under the curve. Briefly it can be considering as the historical record of the collapsed tubes.

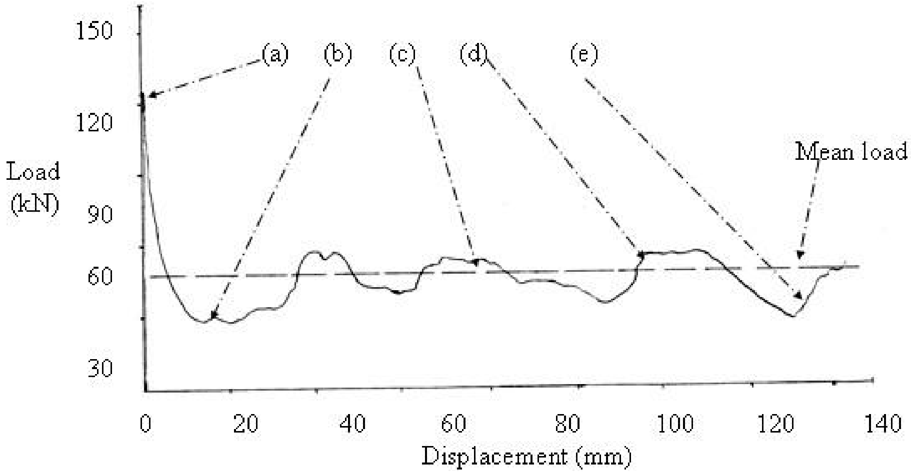


Figure 5: A typical load- displacement curve of the collapsed tube shown in (Figure (3)).

Modes of collapse

From the experimental observations made for the collapsed specimens, four distinct deformation modes were observed as follows:

(a) In-extensional mode of collapse

According to this type of deformation, a collapsing shell contains three sorts of hinges, namely fixed horizontal, fixed incline and travel hinges; see the collapsed specimen in Figure (6). Four horizontal hinges are formed on the opposite sides of each lobes and move outwards, decreasing their initial length; simultaneously, hinges on adjacent sides of lobe move inward, increasing their length. Subsequently, during the collapse process, groups of similar lobes are formed at $\pi / 4$ to previous one and so on. The inclined travelling hinges start at the corners of the tube and finally sweep across the axial length to finish at $\pm 45^o$ to the horizontal ones. It was observed that many specimens tested followed this mode of deformation. This observation is agreed with that made by Johnson et al [4] in similar study.

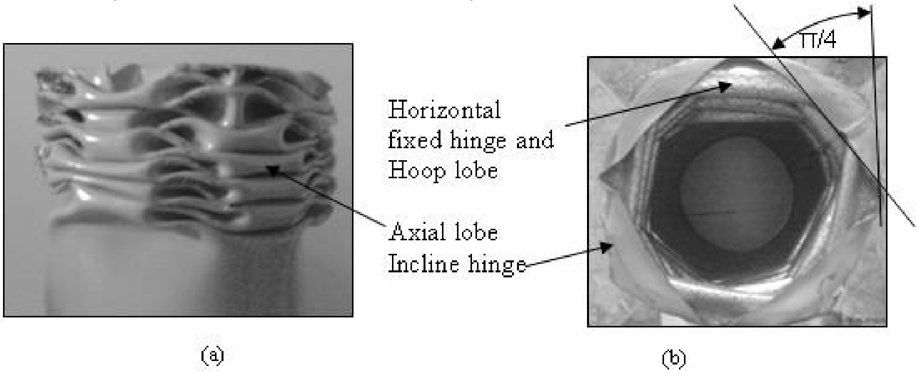


Figure 6: In-extensional collapsed mode (a) Front view, (b) Top view

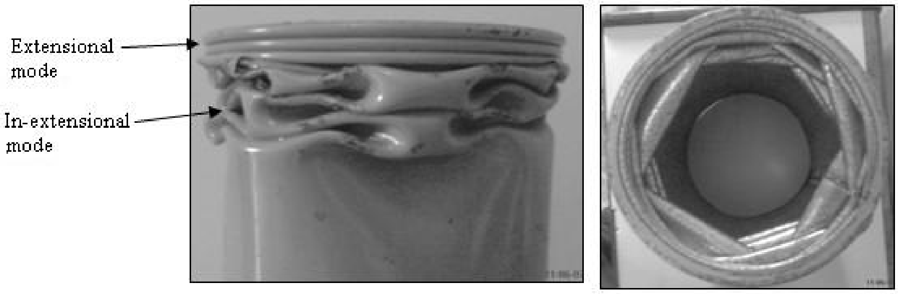
(b) Extensional mode of collapse

This mode of deformation is characterised by formation of circular rings symmetrical about the axis of the shell as shown in Figures (7a - 7b), gives, therefore, the impression of circumferential waves. This mode is similar to that mode which was observed in symmetric ring buckling [3]. Johnson et al [4] observed this mode in their work and they concluded that only very short and very thick tubes failed by this mode because in these particular cases less work required for extending and compressing the tube circumferentially than for folding about including hinges. This conclusion was agreed with observation made about extensional mode in this work.

Figure 7: Extensional collapsed mode. (a) Front view. (b) Top view

(c) Mixed mode of collapse

This type of mode consists of a combination of one or more initially formed symmetric lobes, followed by in-extensional lobes. Figures (8a - 8b) show a collapsed tube exhibited by this mode. It was observed that long tubes usually exhibited this mode by starting to deform from its bottom end by two or three extensible lobes and then the mode attempted to change to the in-extensional mode by initiating a plastic hinges. No analytical information has been found about this type of mode in literature. This manner of collapse probably related to instability of the tube which allows the tube to collapse from the buckled end by less work exhibiting extensible mode. After one or two lobes, the deformed end becomes more stable and rigid to withstand the compression load and attempted to deform by in-extensional mode of collapse. Attempt was made in this study to calculate the stress ratio of this mode.

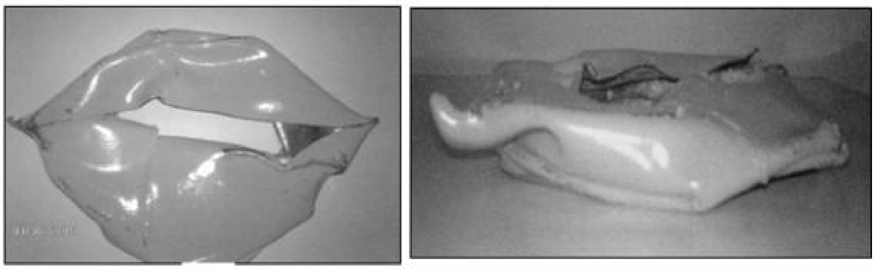


(a) (b)

Figure 8: Mixed collapsed mode. (a) Front view. (b) Top view

(d) Irregular mode of collapse

The tubes in this mode deformed by in-extensional collapse and because of some factors such as variation of the thickness and defects in the materials, the plastic hinges and lobes appear and grow in random manner through the circumferential of the tube. Figures (9a - 9b), shows a collapsed tube exhibited the irregular mode.



(a) (b)

Figure 9: Irregular mode of collapse. (a) Front view. (b) Top view

It was noted that the long tubes exhibited this mode by buckling from its mid length due to instability of the tube under the compression load [11]. To overcome the buckling problem, improve the crushing mode, increase the energy absorption and mean load carrying capacity, such tubes can be filled with light, crushable materials such as polyurethane foam [11]. It was found that the density of the foam played an important factor for energy absorption and mean load carrying capacity.

Theoretical analysis of the collapsed modes

The following analysis is based on the experimental observation of deformation processes which were described in the previous section and the previous studies by Johnson et al [4]. It is assumed that the tube collapses by in-extensional and extensional collapse modes. All deformations being by local bending, the tube ends are free to deform or change shape and the idealized materials of the tube is homogeneous and rigid perfectly plastic.

(a) Plastic hinges

The concept of the work done in the plastic hinges for a collapsed tube can be modeled by a plate with unit width subjects to an axial compression force see Figure (10). At any value of subtended angle (θ), the work done can be indicated by the area under the curve as shown in Figure (11).

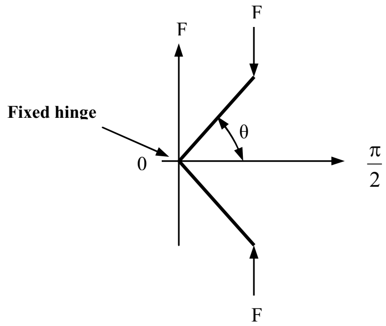


Figure 10: A metal plate under an axial load compressive load

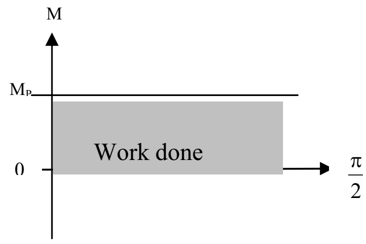


Figure 11: Bending moment-angle curve for the plastic deformation of the plate

This energy dissipated through formation of the high. This concept has been used to evaluate the whole energy absorption, mean load carrying capacity of collapsed tube structures under static and dynamic loading conditions [1-4].

(b) In-extensional mode of collapse

Figure 12 (a) and (b) shows the schematic diagram of the formation of the plastic hinges of the collapsed tubes. Fig.13: shows surface devolvement for four circumference lobes (*n = 4*) and two axial lobes (*m* = 2) for in-extensional collapse model.

Johnson et al [4] demonstrated that the plastic deformation, of the in-extensional collapse mode consists of the following processes:

1-Progressively flattening each curvilinear triangle, i.e. change its radius of curvature from being initially 1/R to infinity and the work done is

$$W_1 = 2\pi\, M_p\, h_o \qquad (1)$$

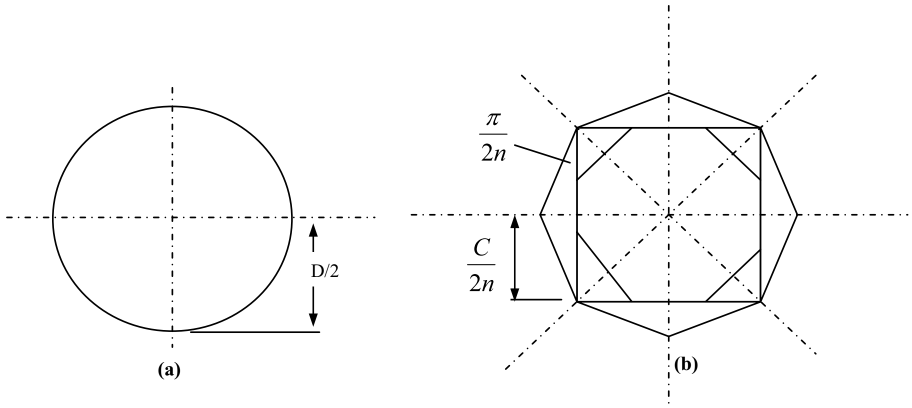


Figure 12: Top plane views of the tube cross-section (a) before and (b) after collapse [4].

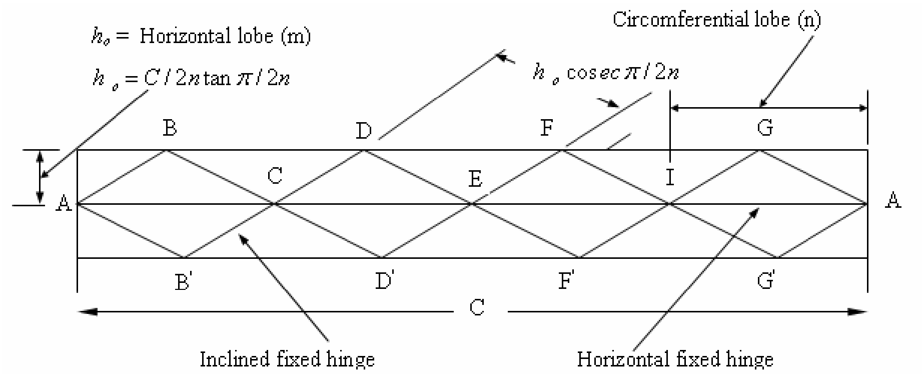


Figure 13: Surface devolvement for four circumference lobes (*n* = *4*) and two axial lobes (*m* = 2) for in-extensional collapse model [4].

2- Bending along each including fixed hinges line. The work dissipated at the including plastic hinges with (*n*) lobes around circumference, the hinge sustain angle, (θ), is $\pi/2n$ and length of $2\,h_o \cos ec(\pi/2n)$ is

$$W_2 = 2\,n\pi\, M_p\ (h_o \cos ec(\pi/2n\ ) \tag{2}$$

3- The fully collapsed tube through the horizontal hinges line ACEIA (see Fig 13). The work dissipated at these plastic hinges (m-1) with (*m*) horizontal lobes is

$$W_3 = \pi\,(m-1)\,M_p\,C \tag{3}$$

4- The work done by the concept of the traveling hinge is quit complex. Johnson et al [4] explained in details the concept of traveling hinges and they deduced following equation:

$$W_4 = 2M_p\,C[\frac{(h_1 - \pi\, r)}{r}] \tag{4}$$

Where: (r) is the radius of travel hinge which can be illustrated in Figure (14). From this Figure, it can be seen that the movement of the hinge as it moves inwards from A_1 to A_2

to A_3. When the hinge reaches the centerline, its motion is restricted by the opposite hinge. As compression continues, the new hinge moves outward until the limit of its travel is reached , when two more stationary bends are formed the process is repeated.

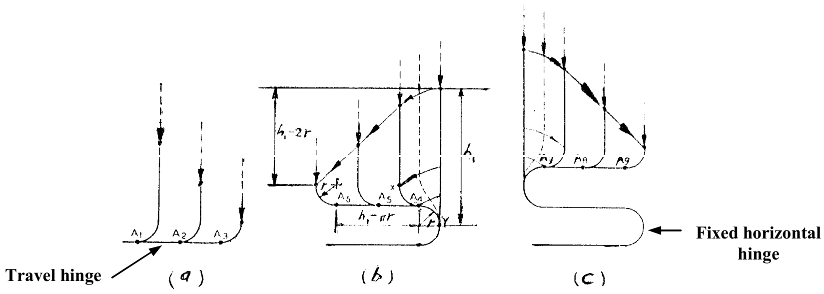


Figure 14: Concept of the traveling hinge and formation of stationary hinges [4]

Thus the total work dissipated by the four types of hinges as:

$$\sum W = W_1 + W_2 + W_3 + W_4 \quad (5)$$

The external work can be estimated [4] by:

$$W_{ext} = \overline{P}(h_1 - 2r) \quad (6)$$

Where the $\overline{P}$ is the mean load carrying capacity and $(h-2r)$ is the effective deformation distance of the tube (see Fig.14). Therefore, equations (5) and (6) must be equal as:

$$\overline{P}(h_1 - 2r) = 2\pi h_1 M_p + 2\pi n M_p \cos ec(\pi/2n) + 2\pi C M_p + 2M_p C(\frac{h_1 - \pi r}{r}) \quad (7)$$

Substitute $C = \pi D = 2\pi h_1 \cos ec(\pi/2n)$ in equation (7) and simplified as:

$$\frac{\overline{P}}{2\pi M_p} = \frac{D/r - n\cot(\pi/2n) + 1 + n\cos ec(\pi/2n)}{(1 - 2r/h_1)} \quad (8)$$

The mean load carrying capacity has minimum value with value of (r^*) [4]. Therefore differentiate equation with respect to r as $\frac{\partial \overline{P}}{\partial r} = 0$. This leads to

$$r^* = \frac{D}{A}(\sqrt{1 + \frac{h_1 A}{2D}} - 1) = \frac{D}{A}(\sqrt{1 + \frac{A\pi}{4n}} \tan(\pi/n) - 1) \quad (9)$$

Where:

$A = 1 + n\cos ec(\pi/2n) - n\cot(\pi/2n)$

By substituting appropriate value of number of lobes *(n)*, the value the value of (*r*) can be determined. Substitute the value of r in equation (8) gives $\frac{\overline{P}}{2\pi M_p}$.

(c) Non-dimensional post-buckling stress

The non dimensional post-buckling stress is defined as the ratio of the mean post-buckling stress to the yielding tensile stress. The mean post-buckling stress is:

$$\bar{\sigma} = \frac{\bar{P}}{\pi D t} \qquad (10)$$

The plastic bending moment per unit length can be calculated as:

$$M_p = \frac{\sigma_o t^2}{4} \qquad (11)$$

By applying Mises yield failure theory for uni-axial loading as:

$$\sigma_o = \frac{2}{\sqrt{3}} \sigma_y \qquad (12)$$

Combined equations (10), (11) and (12) give the non-dimensional post-buckling stress as:

$$\frac{\bar{\sigma}}{\sigma_y} = \frac{1}{\sqrt{3}} \frac{t}{D} \left(\frac{\bar{P}}{2\pi M_p}\right) \qquad (13)$$

Substituting equation (8) into equation (13) leads to the non-dimensional post-buckling stresses for any number of circumferential lobes [4] as:

$$\frac{\bar{\sigma}}{\sigma_y} = \frac{1}{\sqrt{3}} \frac{t}{D} \left(\frac{D/r - n\cot(\pi/2n) + 1 + n\cos ec(\pi/2n)}{(1-2r/h_1)}\right) \qquad (14)$$

(d) Extensional mode of collapse

The collapse mechanism of this mode has been explained by Alexander [3] and modified by Abramowicz and Jones [12]. The folding process is consists of the work done by plastic hinges and work done by stretching the material due to hoop strains. Therefore, the total work done by these two works must equal the mean load carrying capacity multiplied by the total displacement of the collapsed tube. By following the same procedure of the in-extensional mode, Soden et al [13] estimated the non dimensional post buckling stress by the following equation:

$$\frac{\bar{\sigma}}{\sigma_y} = \sqrt{\frac{2\pi}{\sqrt{3}}\left(\frac{t}{D}\right)} \qquad (15)$$

(e) Mixed mode of collapse

The mixed mode is a combination of the extensional and in-extensional modes and therefore the non dimensional post buckling stress equal the summation of the equations 14 and 15 as follows:

$$\frac{\bar{\sigma}}{\sigma_y} = \frac{t}{D\sqrt{3}} \left(\frac{D/r - n\cot(\pi/2n) + 1 + n\cos ec(\pi/2n)}{(1-2r/h_1)}\right) + \sqrt{\frac{2\pi}{\sqrt{3}}\left(\frac{t}{D}\right)} \qquad (16)$$

Effect of the relative length and diameter on the crushing behaviour of the tubes

It was observed that the load-displacement curves of all the tubes exhibit similar characteristics. Figures 15 and 16 show a typical examples of loads-displacement curves for the collapsed tubes of 100 mm diameter and L/D ratio = 1,2,3 and 4.

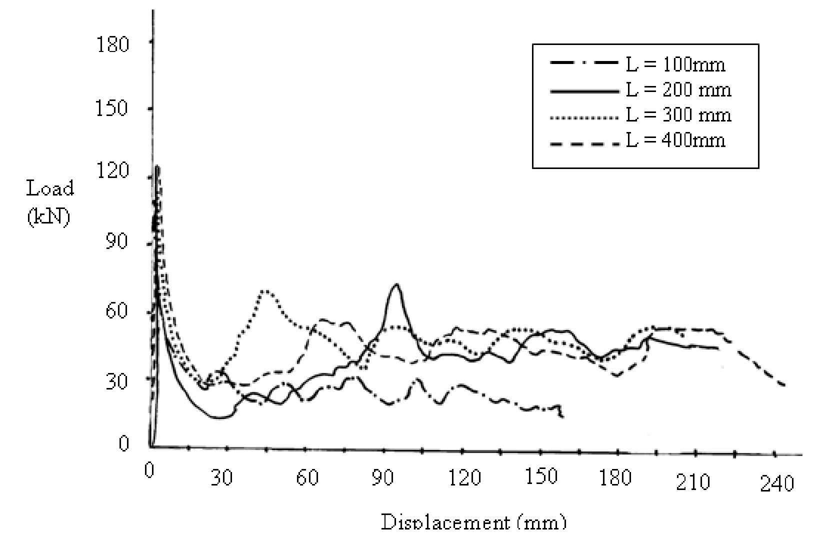


Figure 15: Load- displacement curves for collapsed tubes of L/D = 1,2,3 and 4

It can be seen that the maximum loads (critical loads) are approximately same for all the tubes and the variation of the mean load is not considerable except the tube with L/w = 1 exhibited lower value then the other due to its short length. Energy absorption was found to increase with increase of length for all tubes of diameters 100, 150 and 200 mm. The characteristics of the curves are similar; however, the maximum load carrying capacity is considerably increased with increase of diameter. The energy absorption for short tubes (L/D =1) exhibited similar values whereas these values were found to be increased with increase the diameter for longer tubes.

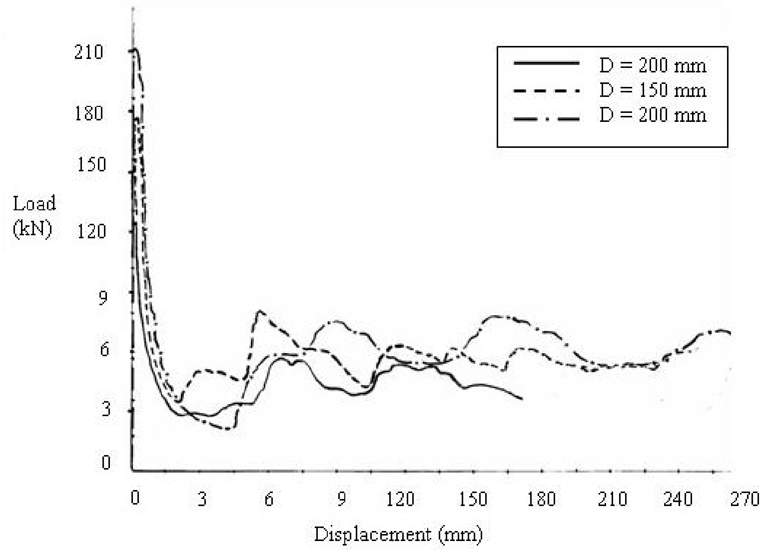


Figure 16: Load- displacement curves for collapsed tubes of different diameters

Figure (17) shows the maximum and main loads carrying capacity versus the length of the tube. It can be demonstrated that the loads are sensibly independent of the length of the tube. In contrast, they increase with increase of the diameter. The energy absorption was found to be increased with increase of the length as shown in the Fig. 18. The tubes of L/D =1 exhibited similar value of energy absorption. These values became different with increase of the diameter, for example, tube of 200 mm diameter with L/D =3 exhibited higher values then the 100mm diameter with same ratio.

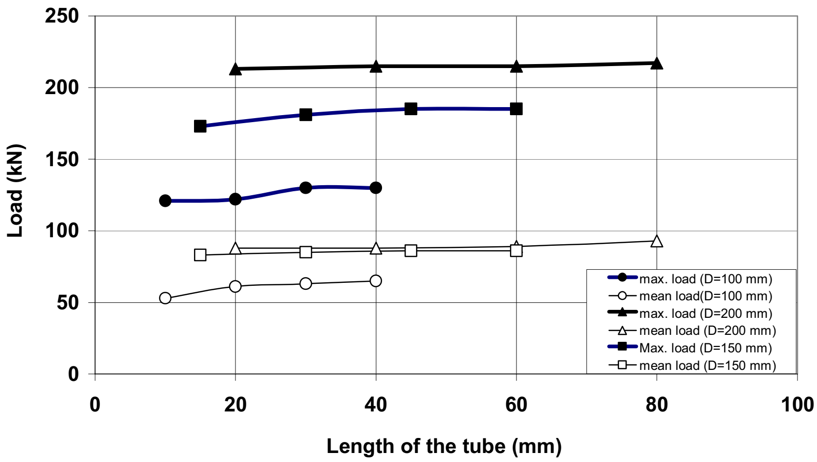


Figure 17: Load carrying capacity versus length of the tubes

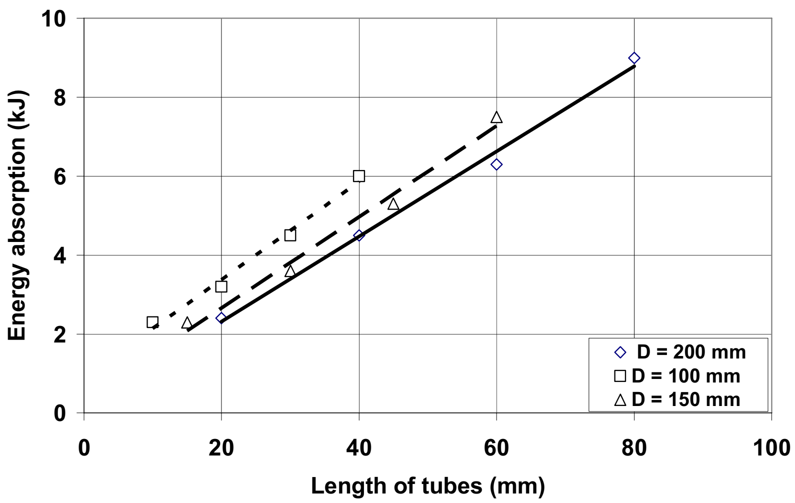


Figure 18: Energy absorption versus length of the collapsed tubes

Effect of the (t/D) on the non dimensional stress ratio

Figures (19 – 20) show the non dimensional stress ratio versus the (t/D) ratio. The stress ratio was derived from equations (14) and (16) for in-extensional and extensional mode of collapse respectively. These expressions are only function of the number of the circumferential lobes (n) and have significant effect on the (t / D) ratio. It can be seen that the stress ratio increases with increase of the (t/D) and number of circumferential lobes. Similar result was observed by Soden at el [13] for crumpling of PVC tubes. They concluded that the energy absorption is directly proportional to (t/D) and it is always lowest for the lowest value of (n).

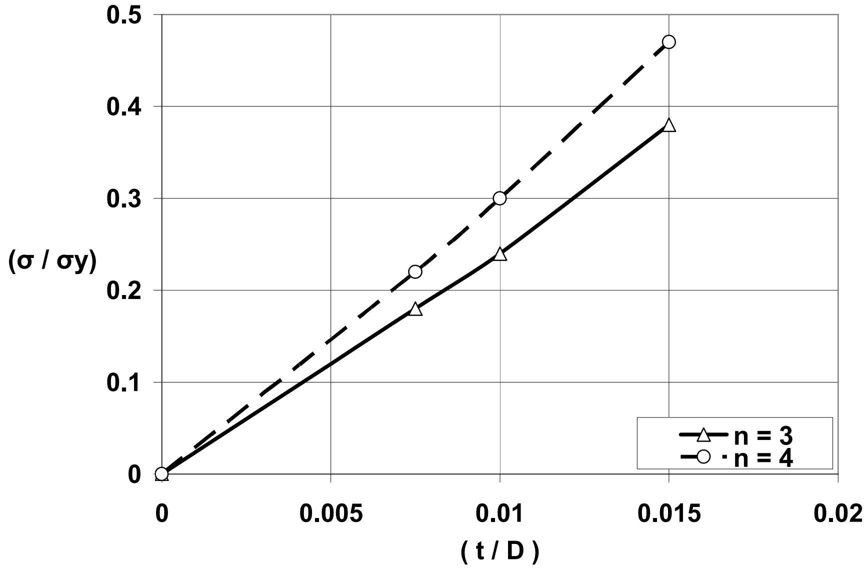


Figure 19: non-dimensional stress versus (t/D) ratio for in-extensional mode of collapse

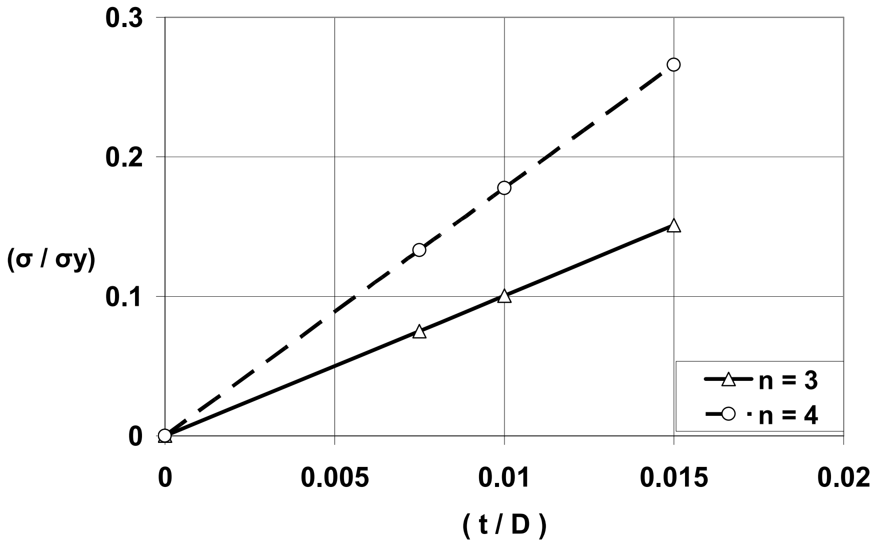


Figure 20: Non-dimensional stress versus (t/D) ratio for extensible mode of collapse

CONCLUSIONS

A number of conclusions are drawn from the above investigations and discussions as follows:

- Four types of collapsed modes have been observed during the crushing of square tubes. These are classified as in-extensible, extensible, mixed and irregular modes.
- The load–displacement curves of all the tubes exhibited similar characteristic and shown an increase in load up to the critical values followed by rapid decrease in load and a series of serrations and peaks about a mean value.
- The maximum and mean loads carrying capacity were found to be independent of the length of the tube.
- The energy absorption was found to be linear depended on the length of the tube.
- The non-dimensional stress was depended considerably on the (t/D) ratio.
- Reasonable agreement between the theoretical and experimental results of mean load carrying capacity (P_{mean}) for the in-extensional collapsed modes was a observed.

ACKNOWLEDGMENT

The authors wish to express their thanks to Engineer. Mohamed Khalil Omran, Engineer Hanan Soliman Mohamed and Mr Mohamed Al-Asmar for their help to carry out the experiment part of this work. The authors are grateful to the Arabian Gulf Oil company for providing the tube specimens. Appreciation and many thanks are extended to the Staff members in Mechanical Department at Garyounis University for their helps.

REFERENCES

[1] Rawlings, B. and Shapland, P., "The behaviour of thin walled box section under gross deformation", The Structural Engineer, Vol. 53, 1975, pp. 181-186,

[2] Meng, Q. Al-Hassani, S.T.S and Soden, P.D., "Axial crushing of Square Tubes" International Journal of Mechanical Sciences, Vol. 25, No. 9-10, 1986, pp. 747-773,

[3] Alexender, J.M., "An approximate analysis of the collapse of thin cylinder cal shell under axial loading", Quarter Journal of Mechanics and applied mathematics, Vol. 13, 1990, pp 11-15.

[4] Johnson, W., Soden, P.D., Hassani, S.T.S, "In-extensional Collapse of Thin-Wall Tube under Axial Compression", Journal of Strain Analysis, Vol. 12, No 4, 1977, PP.317- 330.

[5] Johnson, W., and Reid, S.R., "Metallic Energy Dissipating Systems", Applied Mechanics Reviews, Vol. 31, 1978, pp. 277-288.

[6] Wierzbicki, T. and Abramowicz, W., "On the crushing Mechanics of Thin-Walled Structures", Journal Applied Mechanics, Vol. 50, 1983, pp. 727-734.

[7] Wierzbicki, T. and Huang, J., "Initiation of Plastic Folding Mechanism in Crushed Box Columns", Thin-Walled Structures, Vol. 13, 1991, pp. 115-143.

[8] Marzo, A, Luo, X, Y., Bertram, C.D., "Three-dimensional Collapse and Steady Flow in Thick –walled Flexible Tubes", Journal of Fluids and Structures, Vol. 20, 2005, pp.817- 835.

[9] Hamouda, A.M.S, Saied, R.O. and Shuaeb, F.M., " Energy absorption capacities of square tubular structures", Journal of Achievements in Materials and Manufacturing Engineering, Vol.24, No. 1, 2007, pp36- 42.

[10] ASME Boiler and Pressure Vessel, Section II, Material specification, Part A: Ferrous, 1972 edition.
[11] Reid, S.R., Reddy, T.Y. and Gray, M, D., "Static and Dynamic Axial Crushing of Foam-Filled Sheet Metal Tubes", International Journal of Mechanical Sciences, Vol. 28, 1989, pp. 295-322.
[12] Abramowicz, W. and Jones, N., " Dynamic axial crushing of square tubes", International Journal of Impact Engineering, Vol.2, 1984, pp. 179-208, 198.
[13] Soden, P.D, Al-Hassani, S.T.S and Johnson, J, "The crumpling of polyvinylchloride tube under static and dynamic axial loads", In Mechanical properties at high rates of stain (Edited by Harding J). Institute of Physics, London, Conference Series No 21, 1974, pp 227-337.

NOMENCLATURE

C	Tube circumference
D	Diameter of the tube
R	Tube radius
L	Tube length
T	Tube thickness
h_o	Initial overall height
h_1	Height of one triangular lobe
r	Radii of travelling hinge
r^*	Optimum radius of travelling hinge
$\overline{P}$	Mean collapse load
M	Number of axial lobes
n	Number of circumferential lobes
S	Number of symmetric lobes
M_p	Plastic bending moment
W	Work dissipated in plastic deformation
$\overline{\sigma}$	Mean collapse stress
σ_o	Effective yield strength
σ_y	Yield strength under uni-axial load
C'%	Compression percentage
F	Axial force
θ	Subtended angle