SOLIDFICATION OF PHASE CHANGE MATERIALS ENCAPSULATED IN VERTICAL TUBES

Mabruk M. Abugderah

Faculty of Engineering, Mechanical Engineering Department,
Zawia University, Sabrata Libya
E-mail: mabruk21@hotmail.com

الملخص

تم دراسة تصلب مواد متغيرة الطور مغلفة في اسطوانة عمودية، التي تحتوي كذلك على أنابيب مائع انتقال الحرارة، و ذلك باستخدام نموذج الشبكة الثابتة. يتضمن تحليل المنظومة تبني حقل السرعة للتدفق كامل الطور و قرنه بمعادلات الطاقة للمائع، جدار الأنبوب و مادة تغير الطور. كما تم استخدام طريقة الفوارق المحدودة للحجم المحكوم لحل معادلات منظومة التخزين الحراري المذكورة. تعرض هذه الورقة توزيع درجة الحرارة القطرية، موضع السطح البيني لتغير الطور والحرارة الكامنة و المحسوسة المتراكمة بطول المنظومة كدوال في الفترة الزمنية و رقمي رينولد وستيفان و مواد الأنابيب. كما تعرض هذه الورقة إجراء تنبؤ للأداء الحراري لوحدة حرارية لمادة متغيرة الطور من نوع الانبوب الحراري العمودي.

ABSTRACT

Solidification of phase change materials encapsulated in a vertical shell which contains also the heat transfer fluid tubes is studied using a fixed grid numerical model. The solution of the system consists of adopting a velocity field of a complete developed flow coupled with the energy equations of the heat transfer fluid (HTF), the pipe wall and the phase change material (PCM). The control volume finite difference approach is used to solve the equations describing the phase change thermal system. The radial temperature distribution, the phase change interface position and the latent and sensible heat accumulated along the system axial length in function of time period, Reynolds and Stefan numbers and tube materials are presented. It is also shown in this work a procedure for thermal performance prediction of a vertical tube type phase change material thermal unit.

KEYWORDS: Phase Change Material; Thermal Performance; Solidification Front

INTRODUCTION

Solidification problems are by nature transient ones and, therefore, heat transfer fluid boundary conditions change with interface progress. For this reason, temperature field of the heat transfer fluid would never establish steady state regime before phase change material reaches complete solidification. Due to this fact, this type of problems has limited analytical solutions. Most of the available solutions apply to simplify and idealized one dimensional system. Numerical methods appear to offer a more practical approach to solve a wide range of phase change problems.

Phase change problems have been investigated by various authors taking into consideration thermal and geometrical parameters [1-3]. Elsayed [4] has investigated the periodic melting of encapsulated ice for cold thermal energy storage. A horizontal rectangular container is used as a storage capsule, since warm coolant fluid (glycol) flows over the upper and lower walls of the capsule. The enthalpy method is employed

to model the phase change at the two moving interfaces. The effect of cyclic time variant coolant fluid temperature on the heat transfer efficiency and the corresponding energy stored has been investigated. Also, the influence of the convection heat transfer coefficient and coolant fluid temperature on the melting behavior has been studied and reported. The results show that the coolant fluid temperature affects significantly the melting behavior rather than the convection heat transfer coefficient.

Fang and Chen [5] have published an article in which numerical simulation is carried out to study the effects of different multiple PCMs on the performance of a latent thermal energy storage unit. The numerical analysis is based on the enthalpy change. The numerical results indicate that there is an optimum proportion between the multiple PCMs to obtain the maximum thermal energy charging rate in the latent thermal energy storage unit.

Cao and Faghri [6] solved a phase change thermal storage system conjugated a forced convection. In their work, they defended the importance of the solution of the momentum equations to avoid uncertainties due to the use of empirical relations. Jesus [7] solved the same problem using temperature immobilization method getting satisfactory results.

In this paper, solidification of phase change materials encapsulated in vertical tubes is investigated. First the model is compared with other results obtained from the literature then it is used to analyze a two dimensional phase change storage system in cylindrical coordinates. The influence of various parameters on the performance of the unit; such as the time period, Reynolds and wall materials; is carried out. It is also presented a procedure of a phase change thermal performance prediction.

SYSTEM DESCRIPTION

The system consists of vertical tubes contained in a cylindrical shell, Figure (1). The heat transfer fluid is water and the phase change material is n-Eicosane. The working fluid is flowing in the tubes and the phase change material is located in the shell surrounding the tubes. The chosen PCM is stable chemical, but should be protected from air while hot to preclude slow oxidation. It is not toxic and do not have unpleasant odor. It is combatable with metal containers, is noncorrosive, and is inert. It is flammable, but because of low vapor pressure, present little fire hazard, Lane [9]. Figure (2) illustrates the cylindrical tubes used to carry the heat transfer fluid with a definition of the tube parameters used in the mathematical model. As shown in the figure, the velocity field is assumed to be fully developed.

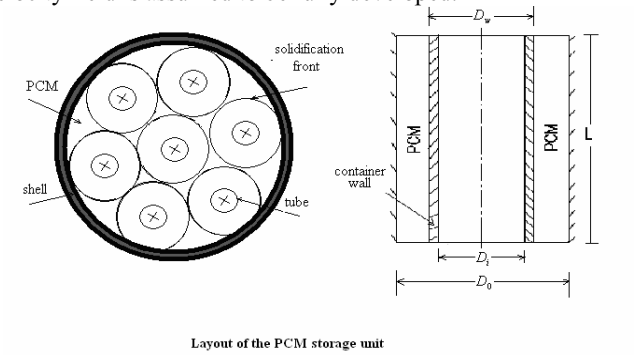

Figure 1: Cross section of the heat transfer system.

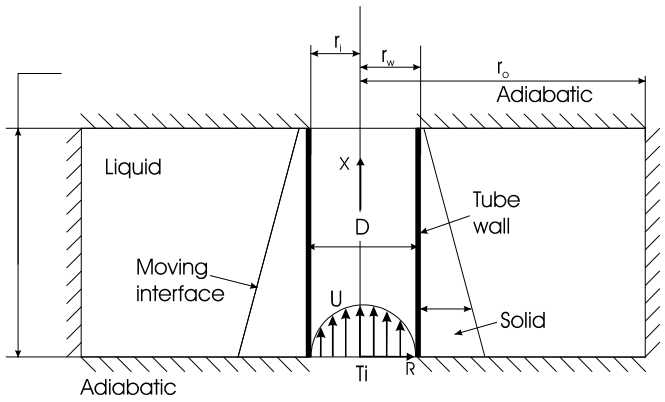

Figure 2: Schematic diagram of the PCM thermal system

MATHEMATICAL MODELLING

The solidification of phase change materials encapsulated in vertical tubes is modelled taking into consideration the initial condition to be of an assumed temperature and the solidification takes place as the heat transfer fluid progresses into the tubes. The solidification process continues until a complete solidification of the phase change material is reached. To study the influence of the parameters involved in the model, the following transformations are adopted:

$$\theta = \frac{T - T_m}{T_{in} - T_m},\ R = \frac{r}{D},\ X = \frac{x}{D},\ U = \frac{u}{U_m},\ \mathrm{Re}_e = \frac{U_m D}{\nu_f},\ \tau = \frac{U_m}{D} t,$$

$$C = \frac{C^o}{c_l},\ K = \frac{k}{k_l},\ St = \frac{c_l \left(T_{in} - T_m\right)}{\lambda},\ \varepsilon = \frac{\delta T}{\left(T_{in} - T_m\right)}$$

where T_m, T_{in}, θ and τ are melting, inlet and non-dimensional temperatures, and non-dimensional time respectively. r and D are radius and diameter of the tube. u, U_m are local and maximum velocities, respectively. Re, ν_f, t, C^o, c_l, k, k_l, St, λ and $2\delta T$, are Reynolds number, working fluid kinematic viscosity, time, thermal capacity, thermal capacity of liquid phase, thermal conductivity and thermal conductivity of liquid phase, Stefan number, latent heat, phase change interface, respectively.

The velocity field, u, of the working fluid is assumed to be fully developed flow and it is given by the equation:

$$U = U_m\left(1 - R^2\right) \tag{1}$$

The dimensionless general energy equation which represents the energy equations of the heat transfer fluid (HTF), the pipe wall and the phase change materials (PCM) is

$$\left(\frac{\partial C\theta}{\partial \tau} + U\frac{\partial \theta}{\partial X}\right) = \xi\left[\frac{1}{R}\frac{\partial}{\partial R}\left(KR\frac{\partial \theta}{\partial R}\right) + \frac{\partial}{\partial X}\left(K\frac{\partial \theta}{\partial X}\right)\right] - \frac{\partial S}{\partial \tau} \tag{2}$$

At the tube wall $U = 0 and C = K = 1$. The term S is given by;

$$S(\theta)=\begin{cases} C_{sl}\varepsilon, & \theta<-\varepsilon \\ C_{sl}\varepsilon+\dfrac{1}{St}, & \theta>\varepsilon \\ \dfrac{1}{2St}+\dfrac{\varepsilon(1+C_{sl})}{2}, & -\varepsilon\le\theta\le\varepsilon \end{cases} \tag{5}$$

and;

$$\xi=\begin{cases} \dfrac{1}{\mathrm{Re}_f\,\mathrm{Pr}_f}, & \text{For the HTF} \\ \dfrac{1}{\mathrm{Re}_f\,\mathrm{Pr}_f}\dfrac{\alpha_w}{\alpha_f}, & \text{For the tube wall} \\ \dfrac{1}{\mathrm{Re}_f\,\mathrm{Pr}_f}\dfrac{\alpha_l}{\alpha_f}, & \text{For the PCM} \end{cases} \tag{6}$$

The thermal capacity of the phase change material is non-dimensional zed in the following form:

$$C(\theta)=\begin{cases} 1, & \theta<-\varepsilon \\ C_{sl}, & \theta>\varepsilon \\ \dfrac{1}{2St\varepsilon}+\dfrac{1+C_{sl}}{2}, & -\varepsilon\le\theta\le\varepsilon \end{cases} \tag{7}$$

In the same manner the thermal conductivity is

$$K(\theta)=\begin{cases} K_{sl}, & \theta<-\varepsilon \\ K_{sl}+\left(1-\mathrm{K_{sl}}\right)(\theta+\varepsilon)/2\varepsilon, & \\ \text{for} & -\varepsilon\le\theta\le\varepsilon \\ 1, & \theta>\varepsilon \end{cases} \tag{8}$$

where $K_{sl}=\dfrac{k_s}{k_l}$, and $C_{sl}=\dfrac{c_s}{c_l}$, k_s, c_s are thermal conductivity and thermal capacity of solid phase.

The non-dimensional form of the initial and boundary conditions is

initial conditions: $\tau=0$

the entire domain: $0\le X\le L/D$; $0<R<R_o$ $\theta=\varepsilon$

boundary conditions: $\tau>0$

entrance conditions: $X=0$; $0<R<R_i$: $\theta=1$

$R_i<R<R_o$: $\dfrac{\partial\theta}{\partial X}=0$

exit conditions: $X = L / D$; $0 < R < R_o$: $\dfrac{\partial \theta}{\partial X} = 0$

outer radius: $0 < X < L / D$, $R = R_o$; $\left.\dfrac{\partial \theta}{\partial R}\right|_{R=R_o} = 0$

fluid-wall interface: $k_w \left.\dfrac{\partial \theta}{\partial R}\right|_{R=R_i^+} = k_f \left.\dfrac{\partial \theta}{\partial R}\right|_{R=R_i^-}$

wall-PCM interface: $k_p \left.\dfrac{\partial \theta}{\partial R}\right|_{R=R_w^+} = k_w \left.\dfrac{\partial \theta}{\partial R}\right|_{R=R_w^-}$

where L is tube length. The subscripts *i, p, f* and *w* are internal, PCM, fluid and tube wall.

Prediction of the thermal performance of a PCM thermal system

Phase change material is specified by the requirement of the project, and therefore it can be determined by its phase change temperature. Taking into account that the thermal properties of the phase change materials are constant, the working fluid inlet temperature can be calculated using Stefan number, which can be specified by a graphical method.

The material of the tube wall is an important parameter in the prediction of phase change system design, due to its effect on the thermal performance and total cost of the equipment. In the selection of this material the following aspects must be considered: it should be an inert material (does not react neither with the working fluid nor with the phase change material), low cost, easy maintenance and high thermal conductivity.

The basic requirement in the prediction of solidification of phase change materials encapsulated in vertical tubes is the specification of the number of tubes to discharge a certain quantity of heat. This type of analysis can be conducted in the following manner: Given or specified parameters are:

- Maximum heat to be discharged by the system;
- Discharge cycle;
- Phase change material.

The Reynolds number can be obtained from a graphical method. Taking into account the physical properties of the working fluid as constant and the Reynolds number is a function of the tube diameter and the working fluid velocity. These two variables can take a range of values for a determined Reynolds number, but the specification of these variables would be tied to the available resources and economical analysis.

The sensible heat discharged from the system is calculated by the relation

$$Q_l = 2\pi D^3 c_p \rho_s \left(T_{in} - T_m\right) \int_{R_i}^{R_o} \int_0^{L/D} \left(\theta - \theta_i\right) R dR dX \tag{9}$$

and the latent heat discharged from the system is calculated by the equation

$$Q_l = \pi D^3 \rho_p \lambda \int_0^{L/D} \left(R_{\text{int}}^2 - R_w^2\right) dX \tag{10}$$

The number of tubes is calculated by dividing the total heat, desired by the phase change thermal system, by the heat stored by one tube.

Numerical Method

The calculation domain is divided into a number of control volumes surrounding each grid point, Figure (3). The general form of the governing differential equation is integrated over each control volume generating a system of algebraic equations. Figure (4) illustrates a control volume for two-dimensional problem where S, N, E, and W are south, north, east and west, respectively.

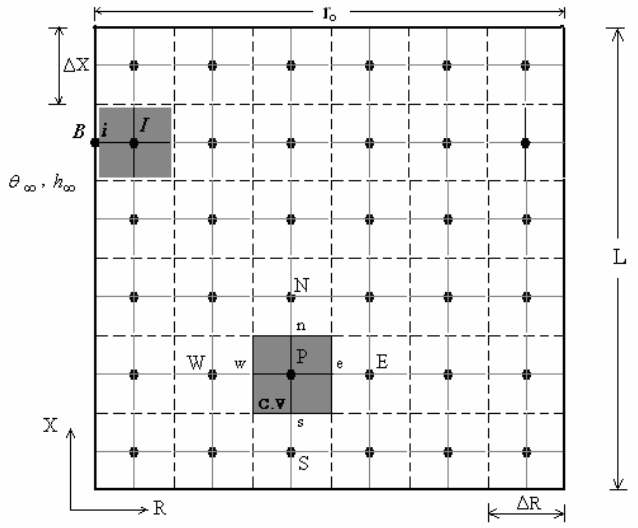

Figure 3: Locations of control volumes.

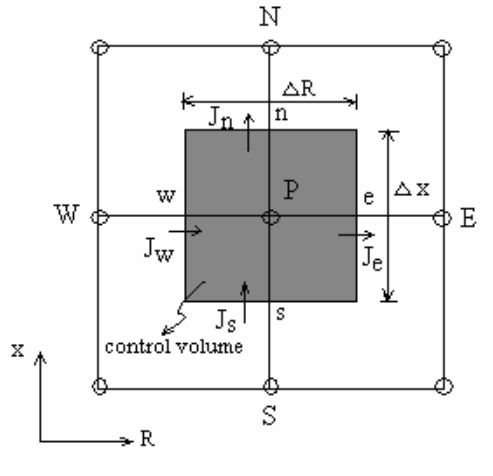

Figure 4: Control volume for two-dimensional problem

The obtained final discretization equation is written as,

$$a_P\theta_P = a_E\theta_E + a_W\theta_W + a_N\theta_N + a_S\theta_S + b \tag{11}$$

where

$$a_E = \frac{1}{R_{grid}}\xi DeA\left(\left|Pe_e\right|\right) + \left\|-Fe,0\right\| \tag{12a}$$

$$a_w = \frac{1}{R_{grid}}\xi DwA\left(\left|Pe_w\right|\right) + \left\|Fw,0\right\| \tag{12b}$$

$$a_N = \xi DnA\left(\left|Pe_n\right|\right) + \left\|-Fn,0\right\| \tag{12c}$$

$$a_s = \xi DsA\left(\left|Pe_s\right|\right) + \left\|Fs,0\right\| \tag{12d}$$

$$a_P = a_E + a_W + a_N + a_S - S_P\Delta V \tag{12e}$$

$$b = S_C^*\Delta V + ap^\circ\theta p^\circ \tag{12f}$$

$$ap^\circ = \frac{C\Delta V}{\Delta\tau} \tag{12g}$$

where. ξ , is defined by Eq.(6).

Here, θ_P° refer to the known value at time (τ) and all other values $(\theta_P, \theta_E, \theta_W, \theta_N \text{ and } \theta_S)$ are unknowns at time $(\tau + \Delta\tau)$.

The flow rates $F_e, F_w, F_n, \text{and } F_s$ are defined as,

$$F_e = V_e A_e = 0 \tag{13a}$$
$$F_w = V_w A_w = 0 \tag{13b}$$
$$F_n = V_n A_n = 0 \tag{13c}$$
$$F_s = V_s A_s = 0 \tag{13d}$$

and their corresponding conductances are defined as,

$$De = \frac{K_e R_e \Delta X}{(\delta R)_e} \tag{14a}$$

$$Dw = \frac{K_w R_w \Delta X}{(\delta R)_w} \tag{14b}$$

$$Dn = \frac{K_n \Delta R}{(\delta X)_n} = 0 \tag{14c}$$

$$Ds = \frac{K_s \Delta R}{(\delta X)_s} = 0 \tag{14d}$$

The Peclet numbers Pe_e , Pe_w , Pe_n and Pe_s are defined by,

$$Pe_e = \frac{Fe}{De} \qquad Pe_w = \frac{Fw}{Dw} \qquad Pe_n = \frac{Fn}{Dn} \qquad Pe_s = \frac{Fs}{Ds} \tag{15}$$

where, $A\left(\left|Pe_i\right|\right) = \left\|0, \left(1 - 0.1\left|Pe_i\right|\right)^5\right\|$ and $i = e, w, n \text{ and } s$

The set of the obtained algebraic equations is solved by the tri-diagonal matrix algorithm (TDMA). The details of the algorithm can be found in Patankar (1980).

RESULTS AND DISSCUTION

Before presenting the numerical results for the phase change thermal system, the phase change model was checked against other numerical results for two dimensional freezing problems in cylindrical coordinates. The obtained results are compared with others obtained by Cao and Faghri [6] who solved a phase change thermal storage system conjugated a forced convection and Jesus [7] who solved the same problem using temperature immobilisation method. The results obtained from the literature together with the results obtained by this work are presented in Figure. (5). As can be seen from the figure, the results obtained by this work are satisfactory. The parameters characterising the problem used for the comparison and used in the work of Cao and Faghri are given in Table (1).

After checking the validity of the model, the numerical calculation for the pcm storage system was then conducted to solve a system consisting of n-Eicosane as the phase change material and water as the working fluid. The melting temperature of n-Eicosane is 36.6 ^{o}C which is suitable for home heating or workshop space cooling. The physical and the system geometrical parameters are given in Tables (2-4).

Table 1: Parameters used by Cao and Faghri [6]

Symbol	Value	Symbol	Value
Re	2200	α_W/α_F	0.11
$St = 0.5$	0.5	k_F/k_W	1.42
Pr	0.065	k_L/k_W	1.42
ε	0.01	r_o/D	1.325
C_{sl}	1	r_W/D	0.575
K_{sl}	1	L/D	12
α_L/α_F	0.02		

Table 2: Physical properties of n-Eicosane

Phase	ρ (kg/m^3)	c_p (J/kg.K)	k (W/m.K)	λ (J/kg)
Solid	778	2010	0.15	247300
Liquid	856	2210	0.15	

Table 3: Physical properties of the working fluid (water)

ρ (kg/m^3)	c_p (J/kg.K)	k (W/m.K)	ν (m^2/s)	Pr
998	4225	0.57	0.012	7.2

Table 4: Operational and geometrical parameters

Tube internal diameter (m)	0.025
Tube wall thickness (m)	0.0015
Tube external diameter (m)	0.028
Tube length (m)	1.5
Working fluid velocity (m/s)	0.25

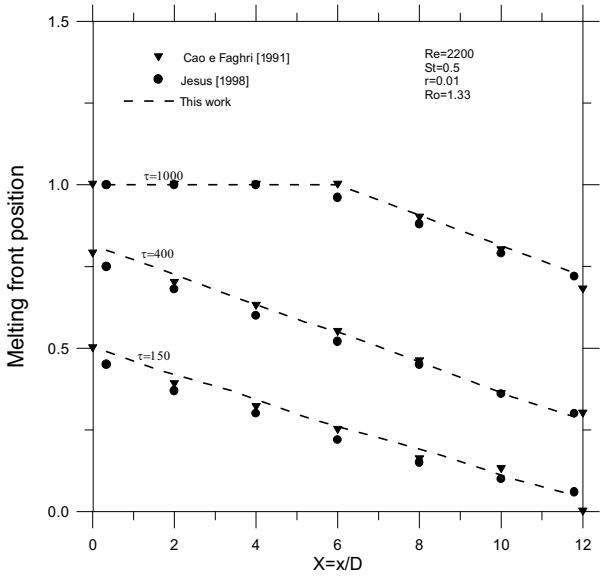

Figure 5: Solidification front position.

Initially the system was considered as liquid at its melting temperature, T_m. The heat transfer fluid inters the tubes with lower temperature than the temperature of the phase change material. The energy of the phase change material is stored as both latent and sensible heat. The stored sensible heat is that used to lower the solid phase temperature to the temperature of the HTF. The grid size used is 60 (axial) × 72 (radial), the last consists of 20 (HTF), 3 (tube wall) and 47 (PCM). A dimensionless length of 60 and time step of $\Delta\tau$, between 20 and 1000 is used. The dimensionless phase change temperature ε is taken to be 0.001, the system initial temperature is $-\varepsilon$, and outer radius r_o/D=2.0.

Figure (6) presents the radial temperature distribution at the middle of the pipe (X=30) for different time periods. The three regions of the domain namely the heat transfer fluid, the pipe wall and the phase change material are illustrated in the figure by the vertical lines. The solidification interfaces of the different time periods are indicated by the intersection of the line passing through θ=1.0 and the corresponding temperature curves. As can be seen from the figure, the temperature curve moves downward and the corresponding melting interface progresses to the right indicating greater energy storage which means that the amount of the solidified material is directly proportional to the time period.

Figure (7), shows the solidification front position along the axial direction at different time periods. As can be seen that at a dimensionless time equal to $2.3\text{x}10^5$ (τ is the given value divided by 100) the solidification interface has reached the outer radius of the system for X<40, while some of the PCM remains liquid for X>40. The reason for that is the high temperature gradient between the heat transfer fluid and the phase change materials which facilitates the heat exchange at the entrance of the system and

consequently decreasing the heat exchange up stream due to the decrease in the temperature gradient between the HTF and the PCM.

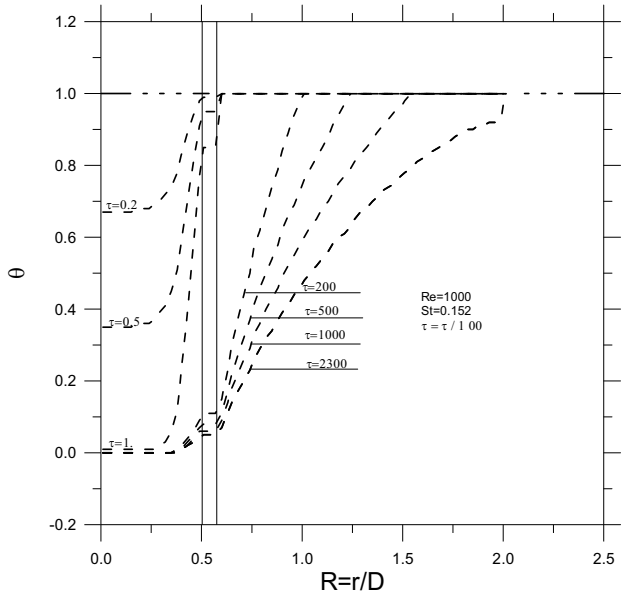

Figure 6: radial temperature distribution at the middle of the pipe.

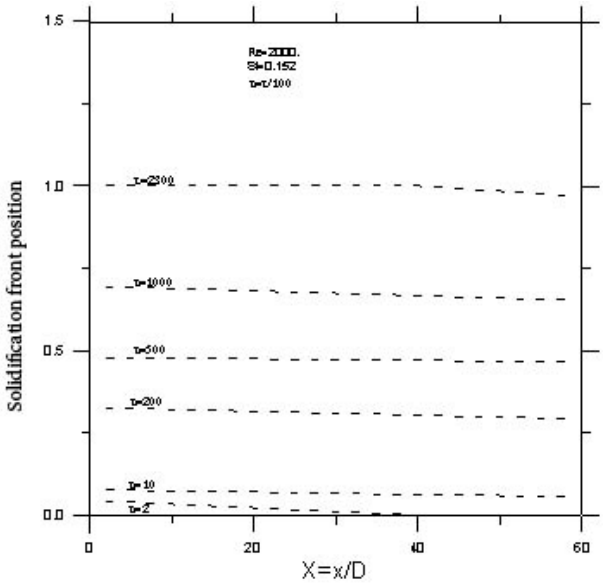

Figure 7: Location of solidification front in the axial direction

Figure (8), shows the solidification front position as a function of the non-dimensional axial length for different Reynolds numbers. It can be seen that for Reynolds number greater than or equal to 1500 the solidification front has reached the outer radius for X<40. It can also be noted that at Re equals to 1500 the solidification front position has reached the outer radius of the phase change materials, i.e., the influence of the Reynolds number is insignificant when passed this value which means that increasing the velocity of the heat transfer fluid will increase the heat exchange between the HTF and the PCM..

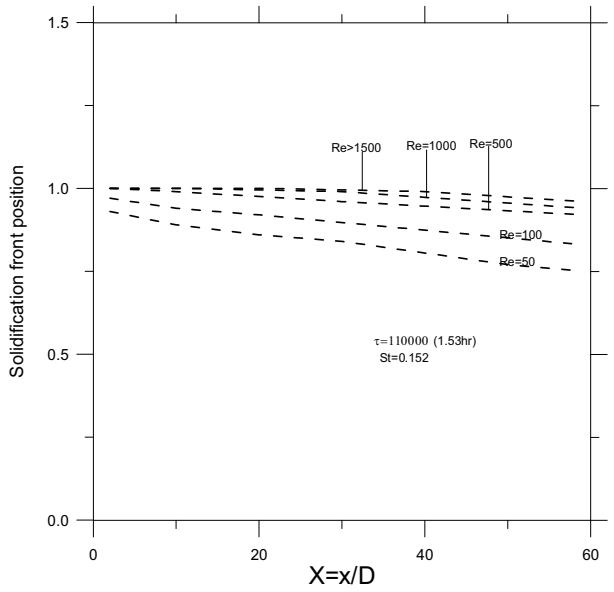

Figure 8: Position of the solidification front for different Reynolds numbers.

Figure (9), shows the influence of the material kind of the tube wall on the solidification front position. As can be seen from the figure, systems with PVC as the wall material gave lower performance than steel or copper that is due to the lower thermal conductivity of the plastic. The identical results of the steel and copper is due the low thermal conductivity of the phase change material compared to the steel and copper and in this way both would have enough capacity to discharge the heat passed from the phase change material to the heat transfer fluid.

Figure (10) presents the time of complete solidification versus the outer radius for different Stefan numbers. The Stefan number in this figure changes only with the inlet temperature of the heat transfer fluid, since, the properties of the HTF are considered to be constant. The Stefan numbers *0.06, 0.15, 0.20, 0.24* and *0.29* correspond to the inlet temperatures *30, 20, 15, 10* and 5^oC, respectively. Observe that the adequate choice of the Stefan number (which would permit the estimation of the HTF inlet temperature) would be about 0.15, because this value corresponds to the ambient temperature and higher value of the Stefan number does not change much the total heat discharged. Other values of the inlet temperature of the working fluid require higher cost. For the

given operational and geometrical parameters, the symmetry radius is found to be about *2.75D*, Figure (10).

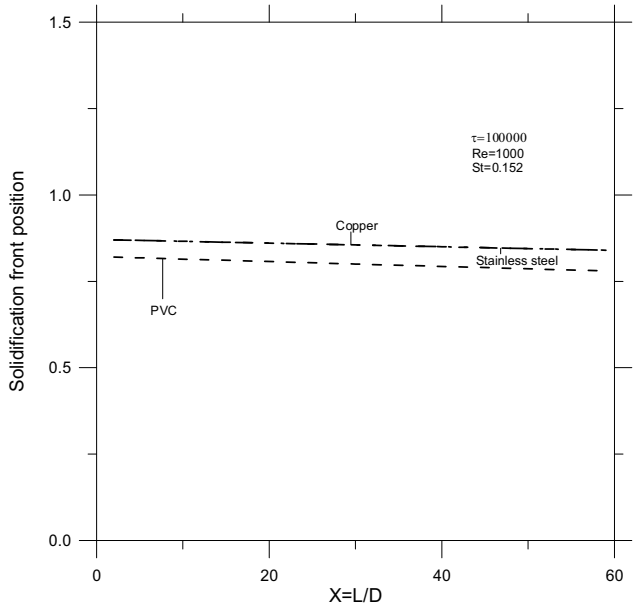

Figure 9: Position of the solidification front for different tube wall material.

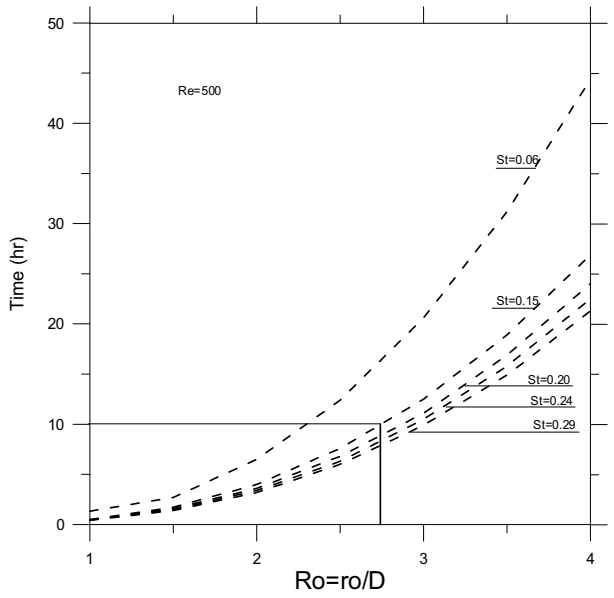

Figure 10: Time cycle versus outer radius for different Stefan numbers

The outer radius is used to estimate the total heat stored by one tube in the system from Figure (11). The sensible heat and latent heat discharged from the system are calculated by Eq. (9) and Eq. (10), respectively.

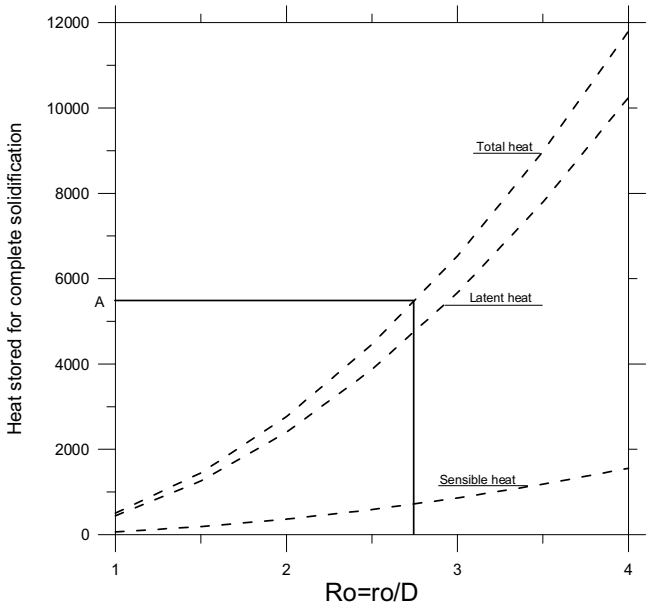

Figure 11: Heat accumulated versus the outer radius of solidification.

CONCLUSION

A solidification phase change materials encapsulated in vertical tubes was investigated using a simplified numerical model where the latent heat was included in a source term and the working fluid flow was considered to be completely developed. The influence of various parameters on the thermal performance of a phase change system was investigated. The most significant parameters are the Reynolds and Stefan numbers, the outer radius of solidification and the tube wall material. The prediction of thermal performance of this type of systems depends on total heat discharged, cycle time, Reynolds number, Stefan number and tube wall materials. Graphs were used to estimate the outer radius of solidification from which the number of tubes necessary to discharge the desired quantity of heat during the cycle time.

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