NUMERICAL ANALYSIS OF THE EFFECT OF SAND MOLD THICKNESS ON THE SOLIDIFICATION TIME OF A PURE MELTED IRON

Giuma M. Fellah and Osama E. Elfituri

Department of Mechanical and Industrial Engineering,
Faculty of Engineering, University of Tripoli
E-Mail: gfellah2008@yahoo.com

الملخص

تم في هذه الورقة تطوير برنامج حاسوب بلغة "فورتران" لتقييم تأثير سُمك القالب الرملي على زمن تجميد الحديد النقي، حيث استخدمت تقنية طريقة الحجوم المحدودة - مخطط الزمن الصريح - طريقة الإنثلبي - لغرض التحليل.

النموذج قيد الدراسة ثنائي البُعد ومنتظم السُمك وتم تثبيت خواص القالب الرملي والحديد مع الأخذ في الاعتبار اختلاف خواص الحديد المصهور عن الحديد الصلب. ويبادل السطح الخارجي للقالب الرملي الحرارة بالحمل الطبيعي مع المحيط الخارجي. وُجد في هذا التحليل أن زمن التجميد يمكن أن يتم التحكم فيه بسُمك القالب الرملي عندما تكون نسبة حجم القالب الرملي لحجم المعدن لا تتعدى مقدار معين، حيث يمكن اعتبارها نسبة حرجة لا يكون بعدها لسُمك القالب الرملي أي تأثير على زمن التجميد.

ABSTRACT

A computer program in FORTRAN is developed to evaluate the effect of sand mold thicknesses on the solidification time for pure iron. The finite volume technique explicit time scheme, enthalpy method is adopted for the analysis.

The adopted sand mold is a two-dimensional one of uniform thickness, the thermo-physical properties of the mold and the metal are assumed to be temperature independent, however, for the metal, the properties can be different in the solid and liquid phases. The outer surfaces of the mold transfer heat by convection to surroundings.

It is found that the solidification time can be controlled by the mold thickness only when the ratio of the sand size to metal size is smaller than or equal to a definite value. This ratio can be considered as the critical one, beyond which the mold thickness has no effect on the solidification time.

KEYWORDS: Phase Change Materials; Moving Boundary Problems; Mold; Finite Volume technique; Enthalpy Method.

INTRODUCTION

Currently, the foundry industry approximates that about 28% of sands are directed to useful use. The American Foundry Society has set a goal of 50% useful use by 2015. As the valuable use of the foundry sand grows to be a key challenge, using the appropriate size of a mold to solidify a given amount of molten metal could be a crucial issue from the economic and environmental point of view. About 9-10 million tons of mold sand are leftover by foundries in the United States each year [1].

Phase change problems play the key role in the foundry industry, where melting and solidification involving mass and heat transfer [2, 3]. The main confront of such problems is the existence of a moving solid-liquid interface involving a strong coupling of heat and mass transfer [2].

With the increasing attention to evaluating the solidification of foundry pieces, numerous methods have been used throughout the years. Both, analytical and numerical techniques have been adopted [4].

Phase change problems are non-linear due to the existence of a moving interface, hence, few analytical solutions can be found except for simple cases, which are of limited useful concern. Therefore, numerical techniques deal with complex, multi-dimensional problems.

Heat conduction mode is considered for solidification simulation. The heat source in the solidifying liquid is the latent heat of fusion liberated progressively as the solidification proceeds [5]. Since solidification is basically the end result of heat transfer throughout the process, the thermal properties of the mold material, like thermal conductivity and diffusivity affect the total solidification time and the cost of the productive process [6].

The effect of natural convection on the total solidification process of squeeze casting is presented by A. Yu et al. [7]. R. Laqua et al. [8] presented a simulation of mold filling and solidification of investment casting processes by using the CFD program FLUENT and the casting simulation tool C. Hernan[9] developed a numerical model to analyze the melting solidification process considering the natural convection phenomena. I. Nova[10] studied the thermal processes in the casting by using SIMTEC program to simulate the solidification process for the sand mold system and comparing numerical results with experimental data. M. Gonzalez et al. [11] developed a computational simulation system for modeling the solidification process in a continuous casting for steel slabs. S. Yoo et al. [12] employed numerical simulations to optimize the casting design and conditions for large cast iron castings for marine engines.

ANSYS software has been used to obtain the temperature distribution in the casting process by performing Transient Thermal Analysis [13]. Results obtained by simulation software are compared with the experimental reading of temperature and found to be in good agreement.

An experimental exploration was carried out to evaluate the rate of solidification of commercially pure aluminium in metallic moulds [14]. They reported that the solidification time increases by increasing the mold thickness.

In the present work, the effect of mold thickness on the solidification time of pure iron is presented. The analysis is based on the enthalpy method, and the problem is considered as two-dimensional transient heat conduction.

THE MATHEMATICAL MODEL

Solidification is characterized by phase change and liberation of latent heat at the solid-liquid interface. Figure (1) depicts the schematic diagram of the system geometry.

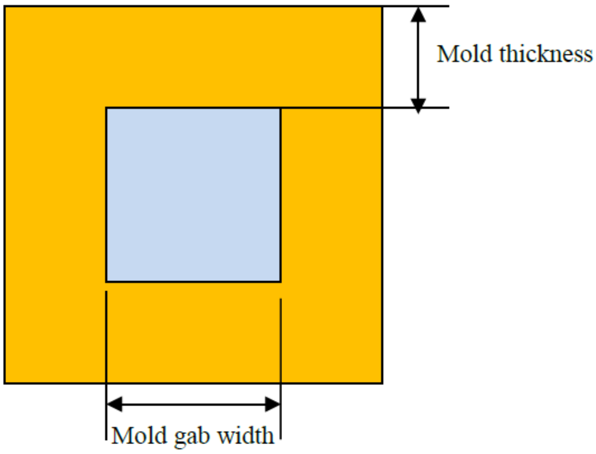

Figure 1: The system geometry

The governing differential equation for unsteady, two-dimensional conduction heat transfer is [15]:

$$\rho_p \frac{\partial I}{\partial t} = k_P \left[\frac{\partial^2 T_p}{\partial x^2} + \frac{\partial^2 T_p}{\partial y^2} \right] \qquad (1\text{-a})$$

Boundary conditions:

The boundary conditions are of the convection type such that at the left and right faces can be written as:

$$h(T_{amb} - T_W) = -k_P \frac{\partial T_P}{\partial x} \qquad \text{for} \quad t > 0 \qquad (1\text{-b})$$

and at the bottom and upper faces as:

$$\mathrm{h}(\mathrm{T_{amb}} - \mathrm{T_W}) = -\mathrm{k_P} \frac{\partial \mathrm{T_P}}{\partial \mathrm{y}} \qquad \text{for} \quad t > 0 \qquad (1\text{-c})$$

Initial conditions

The initial condition in the metal region is:

$$T_P(x, y) = T_{mt} \qquad \text{at} \ \ t = 0 \qquad (1\text{-d})$$

and for the sand mold is:

$$T_P(x, y) = T_{mold} \qquad at \ \ t = 0 \qquad (1\text{-e})$$

and in the liquid phase:

$$T_P(x, y) > T_m \ k_P = k_l, \ \rho_\mathrm{P} = \rho_\mathrm{l} \qquad (1\text{-f})$$

and in the solid phase:

$$T_P(x,y) < T_m \ \mathrm{k_P} = \mathrm{k_s}, \rho_\mathrm{P} = \rho_\mathrm{s} \tag{1-g}$$

Finally, in the sand region:
The enthalpy is determined as:

$$I = \begin{cases} C_P(T - T_m) & for \quad T < T_m \\ C_P(T - T_m) + L & for \quad T > T_m \end{cases} \tag{2}$$

Conversely, given the enthalpy of substance, the corresponding temperature is determined as:

$$T = \begin{cases} T_m + \dfrac{I}{C_P} & for \quad I < 0 \\ T_m & for \quad 0 \le I \le L \\ T_m + \dfrac{I - L}{C_P} & for \quad I > L \end{cases} \tag{3}$$

$$k_P = k_{mold}, \ \rho_\mathrm{P} = \rho_\mathrm{mold}, \qquad \text{and} \qquad \mathrm{I}(\mathrm{x,y}) = C_{P\,mold}\left[T_P(x,y) - T_m\right]$$

VALIDATION OF THE MODEL

The analytical solutions for moving boundary problems are applicable only in limited cases. They are mainly for one-dimensional cases of the semi-infinite region with simple initial and boundary conditions and constant thermal properties. The presented system is validated by using one-dimensional, semifinite, long slab, analytical solution. The numerical model is modified to suit the analytical model. The boundary condition is imposed as a constant temperature at one side, and the size of the system is increased, such that a semi-infinite system prevails. The slab is a 0.15 m long at an initial temperature of 1923K. The boundary condition at one end is at a constant temperature of 300K. The properties of solid and liquid phases are given in Table (1) [12].

Table 1: Physical properties (pure iron and sand)

Properties	Sand	Iron	
		Liquid	Solid
K, Wm^{-1} K^{-1}	1.5	22	78.2
ρ, kg m^{-3}	1600	7015	7870
C$_p$, kJ kg^{-1} K^{-1}	1.117	0.795	
L, J kg^{-1}		272.090	
T, K		1812	

Table (2) shows the numerical solution of the starting and endingtime for complete solidification. While Table (3) compares the position of the interface, both for the numerical and analytical solutions, good agreement is found. The results are plotted in Figure (2).

Table 2: Numerical solution of the starting and ending time for complete solidification

Distance x (m) from right side	starting time (s)	ending time (s)
0	0	0
0.005	1	2
0.015	5	8
0.025	12	18
0.035	22	31
0.045	36	48
0.055	53	68
0.065	74	93
0.075	99	121
0.085	127	153
0.095	160	189

Table 3: Interface position

Time (sec)	Numerical interface position at solidification starting time	Numerical interface position at Solidification ending time	Average Numerical interface position	Analytical interface position
0	0.00	0.000	0.000	0.000
1	0.005	0.0025	0.00375	0.00373
5	0.015	0.0100	0.0125	0.0163
12	0.025	0.0190	0.022	0.0253
22	0.035	0.0281	0.03155	0.0342
36	0.045	0.0379	0.04145	0.0438
53	0.055	0.0475	0.05125	0.0531
74	0.065	0.0574	0.0612	0.0627
99	0.075	0.0671	0.07105	0.0726
127	0.085	0.0769	0.08095	0.0822
160	0.095	0.0869	0.09095	0.0923

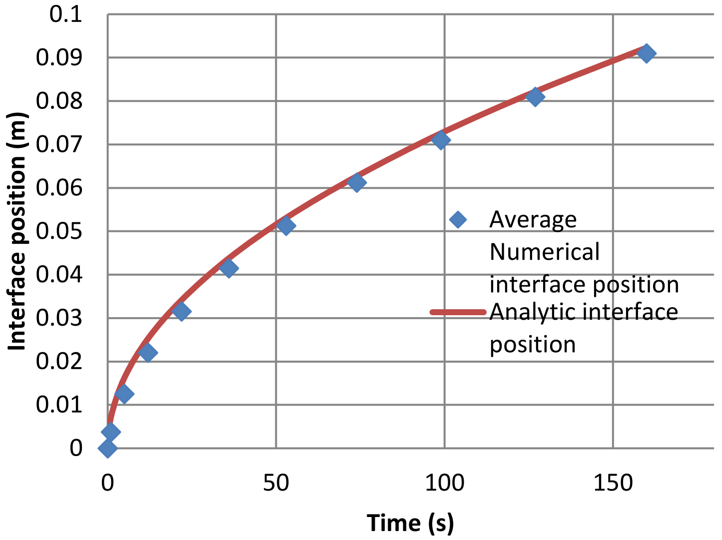

Figure 2: Interface position

RESULTS AND DISCUSSION

The present work explores the effect of sand mold thickness on the solidification time of a melted pure iron. The temperature distributions inside iron and sand moldarenumericallycalculated. A reliable and practicalsand mold gap dimensions of 26 cm × 26 cm, and a sand thickness of 12 cm are selected for simulation. The pure iron is initially in the liquid phase at 1923K (the melting temperature of pure iron ‘Fe’ is 1538^{o}C). The initial temperature of the sand mold is 300K.A typicalvalue of the free convection heat transfer coefficient on the external surface of11.45 W/m^{2}.K is selected. The temperature distribution contours for time periods of 0.5 and 1 hours are numerically calculated and plotted in Figure (3) and Figure (4), respectively. It can be shown that the interface temperature of the metal and sand reaches 1200^{o}C and 1600^{o}C after 0.5 and 1.0 hours.

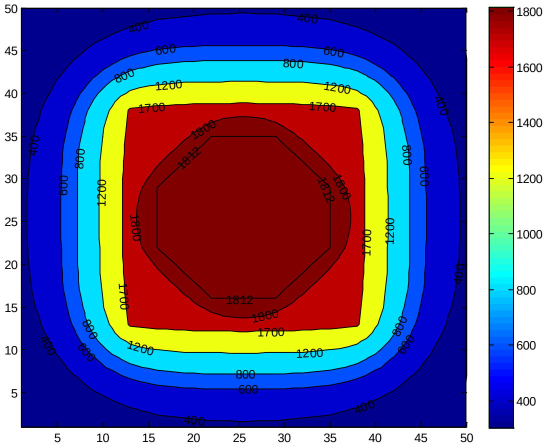

Figure 3: Temperature contour after 0.5 hr

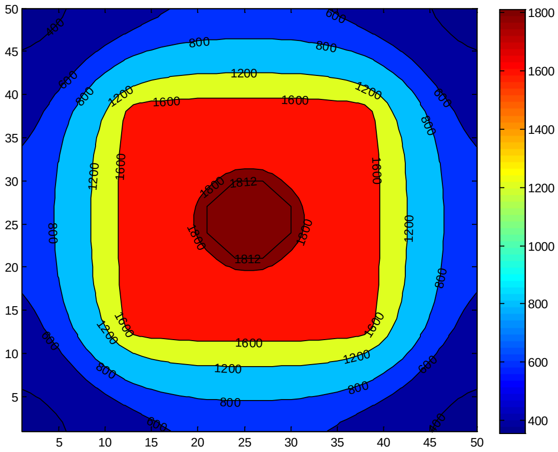

Figure 4: Temperature contour after1 hr

Figure (5) shows the temperature distribution contours for complete solidification after 1.22 hours. It can be seen that the interface temperature reaches the melting temperature of the pure iron. The discrepancy in temperature gradient between iron and sand is due to the differences in the thermal diffusivities (for iron = 218×0^{-7} m^{2}/s, and for sand = 8.5×10^{-7} m^{2}/s).

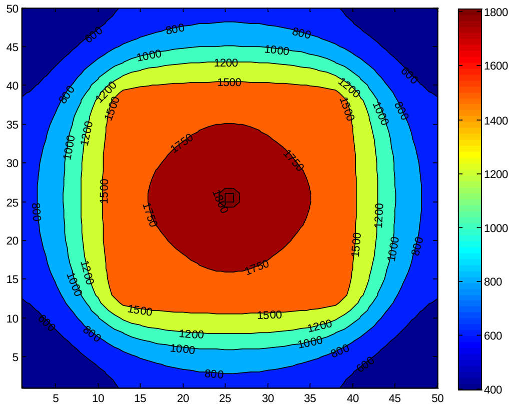

Figure 5: Temperature contour after 1.22 hr (complete solidification)

Figure (6) presents the solidification time for different iron metal size versus sandthickness.The sandthickness has a little influence on solidification time for small metal sizes. However, for larger sizes, the effect of sand thickness is noticeable up to certain value after which the variation of solidification time with sand thickness vanishes. It is found, a minimum amount of mold material is necessary to absorb most of the energy liberated by the metal, beyond that, the sand mold thickness has no influence on the solidification time. An excessive amount of mold material might not be required to speedupthe solidification process. Economically, this result is vital, as the minimum amount of mold material should be determined for the given size of the molten metal.

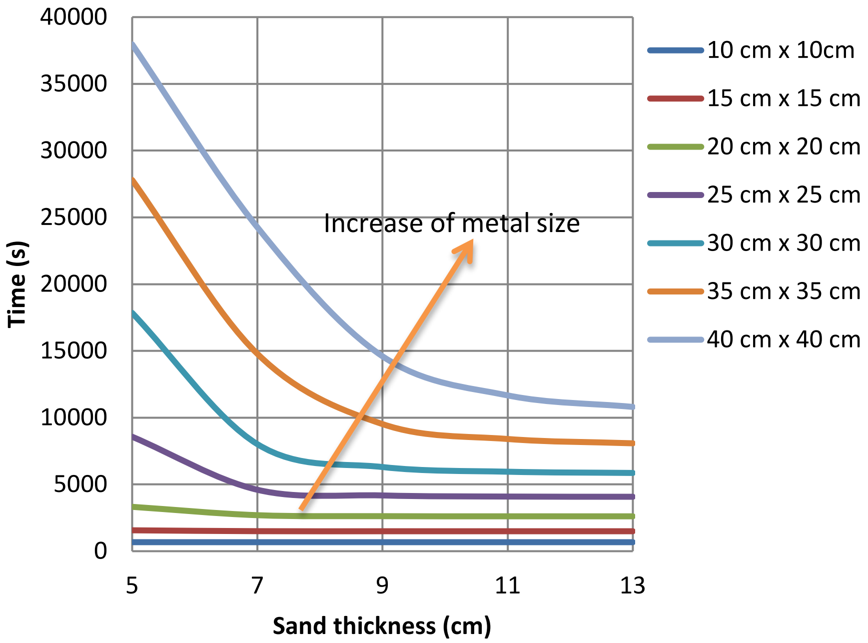

Figure 6: End of solidification time for different iron metal size with variation of sand thickness

To explore the results, the ratioofthe absorbed heat by the sand mold to that released by the molten metal is plotted against the mold sand thickness for two metal sizes, as shown in Figure (7). The mold thickness does not affect the melting time, as the heat ratio becomes invariant with an increase in the mold thickness. For small metal size (15×15 cm), the heat ratio is almost constant even for relatively small mold thickness (5 cm), this result can be interpreted as 98% of the released heat by the metal is absorbed by the sand, and when the sand thickness reaches 9 cm the heat ratio becomes 100%, that is, all heat released by the molten metal is absorbed by the sand material and the problem can be treated as a semi-infinite problem. For larger size (30×30 cm), it can be seen that there is a steep increase in the heat ratio up to sand thickness of about 7 cm, after which the heat ratio becomes less steep, and for sand thickness larger than 11 cm, the heat ratio becomes almost of constant value of 97%, that mean about 3% of the released heat is convected to the environment by the external sand mold surface area.

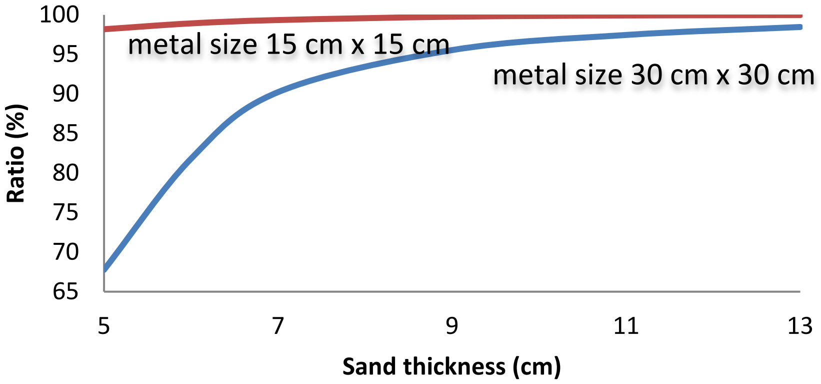

Figure 7: Percentage of the heat energy storage in the sand mold

The end of solidification time is plotted against the ratio of mold size to metal size for different metal sizes in Figure (8).

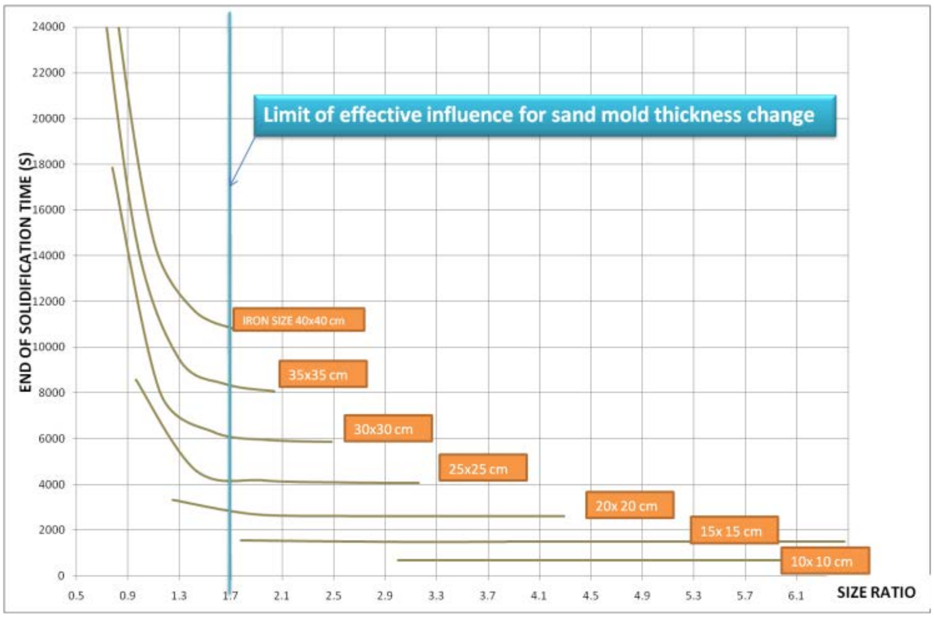

Figure 8: End of solidification time for different iron metal size with sand Iron size ratio

The influence of size ratio on the melting time is limited for relatively small metal sizes. However, for relatively large metal size, there is initially a sharp decrease in the solidification time with the increase in size ratio, where a critical size ratio exists for each metal size beyond which the melting time becomes invariant and the mold size has no influence on the end of the solidification time. A critical ratio of about 1.7 can be approximated for most metal sizes considered in this simulation.

CONCLUSIONS

The process of solidifying of pure iron metal is simulated, where the effect of varying sand mold thickness on the solidification time is explored. Results show a direct link between the thickness of the metal size, sand mold thickness and the solidification time.

In the current simulation, it is found that the end of the solidification time can be controlled by the sand-mold thickness up to the point where the mold size to metal size equal to 1.7. The exact value of the mold thickness, which is required to complete the solidification process for a given metal size, is very important from the economical and environmental point of view.

REFERENCES

[1] USEPA, Actions aimed at increasing the beneficial use of foundry sand a multi-stakeholder action plan, p. 16, 2009.

[2] Y. Belhamadia, A. Kane, and A. Fortin, An enhanced mathematical model for phase change problems with natural convection, *Int. J. …*, vol. 3, no. 2, pp. 192–206, 2012.

[3] C. Guha, Numerical Analysis of Solidification of Sulphur Over a Moving Surface, no. I, pp. 22-27, 2007.

[4] R. Ii, Simulation of the aluminum alloy A356 solidification cast in cylindrical, pp. 294-303, 2008.

[5] O. O. Oluwole, G. O. Oluwadare, and A. A. Afonja, Development of Mathematical Equation for Fraction of Solid in AlSi7Mg Alloy Solidification Simulation, vol. 7, no. 1, pp. 71–82, 2007.

[6] A. C. Mossi, Numerical Simulation of Heat Transfer During the Solidification of Pure Iron in Sand and Mullite Molds, vol. XXVII, no. 4, pp. 399-406, 2005.

[7] A. Yu, N. Li, H. Hu, and F. M. Company, Numerical simulation of natural convection in magnesium alloy squeeze casting, pp. 119-126, 2003.

[8] R. Laqua, T. Ivas, J. Scheele, and J. Jakumeit, Mold Filling and Solidification Simulations of Investment Casting Processes using CASTS-FLUENT, pp. 1-9, 2003.

[9] C. H. Salinas Lira, Alloy Aluminum Solidification in Square Section, *Rev. Fac. Ing. - Univ. Tarapacá*, vol. 14, no. 1, pp. 16-25, 2006.

[10] I. Nova, Solution of the processes of solidification and cooling of the castings in sand moulds by means of simulation calculations, Archives of foundry, vol. 2, no. 4, 2002.

[11] M. Gonzalez, M. B. Goldschmit, A. P. Assanelli, E. N. Dvorkin, and E. F. Berdaguer, Modeling of the solidification process in a continuous casting

installation for steel slabs, *Metall. Mater. Trans. B*, vol. 34, no. 4, pp. 455–473, 2003.

[12] S. M. Yoo, Y. S. Cho, C. C. Lee, J. H. Kim, C. H. Kim, and J. K. Choi, Optimization of Casting Process for Heat and Abrasion Resistant Large Gray Iron Castings, *Tsinghua Sci. Technol.*, vol. 13, no. 2, pp. 152-156, 2008.

[13] C. M. Choudhari, B. E. Narkhede, and S. K. Mahajan, Modeling and Simulation with Experimental Validation of Temperature Distribution during Solidification Process in Sand Casting, vol. 78, no. 16, pp. 23–29, 2013.

[14] K. C. Bala, R. H. Khan, M. S. Abolarin, and O. K. Abubakre, Investigation on the Rate of Solidification and Mould Heating in \nthe Casting of Commercially Pure Aluminium in Permanent Moulds of varying Thicknesses \n, *IOSR J. Mech. Civ. Eng.*, vol. 6, no. 1, pp. 33-37, 2013.

[15] M. N. Ozisik, Heat conduction, 2nd Edition, John Wiley and Sons, New York, 1993.

NOMENCLATURE

A	cross-section area, m^2
C_p	specific heat at constant pressure, J/kg ºC
h	heat transfer coefficient, W/m^2 ºC
I	specific total enthalpy, J/kg
k	thermal conductivity, W/mºC
L	specific latent heat, J/kg
q	heat transfer rate, W
t	time, s
T	temperature, ºC
x, y	Cartesian coordinates, m

Greek letters:

α	thermal diffusivity, m^2/s
ρ	density, kg/m^3

Subscripts:

amb	ambient
i	initial
l	liquid
m	melting
mt	metal
P	node center
s	solid
w	wall