HEAT TRANSFER ENHANCEMENT IN TURBULENT FLOWS UTILIZING NANOFLUIDS

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الملخص

تقدم هذه الورقة دراسة تحليلية لمحاكاة انتقال الحرارة بالحمل القسري والإجهادات القطعية للتدفق تحت ظروف التدفق المضطرب داخل أنابيب معرضة لتسخين منتظم على أسطحها باستخدام الموائع النانوية التي تتكون من خليط من مائع أساسي وجسيمات صلبة نانوية. تم دراسة عدة أنواع من الموائع النانوية المشتملة على الموائع الأساسية من الماء أو الإيثلين جيلي كول أو زيت المحرك وجسيمات النانو المخلوطة بالموائع على أكسيد الألومنيوم أو النحاس أو أكسيد التيتانيوم. تم اعتبار الخليط كطور واحد وبذلك تمّ إهمال السرعة الانزلاقية بين الموائع والجسيمات وكذلك تمّ اعتبار في التدفق وأنّ الخليط متزناً حرارياً. لتحديد الفيض الحراري والاجهادات للموائع النانوية في التدفق المضطرب تمّ استخدام معادلات رينولدز المتوسطة لمعادلات نافيير –ستوكس ونموذج كي –إبسلن المنطرب تمّ استخدام معادلات رينولدز المتوسطة لمعادلات نافيير مونوذج كي السلام المنطرب تمّ استخدام معادلات وينولذ المتوسطة لمعادلات الفير المنطرب تمّ استخدام معادلات وينولذ المتوسطة لمعادلات نافيير –ستوكس ونموذج كي –ابسلن المنطرب تمّ استخدام معادلات رينولدز المتوسطة لمعادلات الموائع النانوية في التدفق المنطرب تمّ استخدام معادلات وينولذ المتوسطة لمعادلات نافير مونوذج كي السلام المنطرب تمّ استخدام معادلات وينولذ المتوسطة لمعادلات نافير الموائع النانوية في التدفق المنطرب تمّ الموزية الحراري عند 2000 وات/م². أمّا رقم رينولدز فقد تمّ تغييره بين 500 مالتملة المنطرب. حيث كانت ظروف التسخين عند ثبوت درجة الحرارة عند 333 كلفن على سطح الأنابيب وعند ثبوت الفيض الحراري عند 5000 وات/م². أمّا رقم رينولدز فقد تمّ تغييره بين 500 مالي التدفق المحسين انتقال الحرارة وانتقال الحرارة وكما أنها مناسبة جداً النانوية لها إمكانات كبيرة على تحسين انتقال الحرارة وانتقال الحرارة وكما أنها مناسبة جداً التطبيقات الصناعية والعملية.

ABSTRACT

A numerical simulation of forced convective heat transfer and wall shear stress under turbulent conditions inside a uniformly heated tube using nanofluids is presented in this paper. The nanofluid is a solid-liquid mixture in which metallic or nonmetallic nanoparticles are suspended in a base fluid. Water, ethylene glycol and engine oil are used as base fluids. Nanoparticles of AL_2O_3 , Cu and TiO_2 are used as mixtures of nanofluids. The solid-liquid mixture was considered as a single-phase, so the slip velocity between the phases was neglected. Moreover, symmetry in flow and local thermal equilibrium of the mixture were assumed. To define heat flux and stresses of the nanofluids in turbulent flow, the Reynolds-averaged Navier-Stokes equation and k- ε turbulent model were used. The wall conditions were constant wall temperature at 333 K and uniform heat flux of 50,000 W/m² and the Reynolds number was varied in the range of 5000-30,000. The effect of nanoparticles concentration, material and base fluid type on heat transfer enhancement and pressure drop were investigated. The results showed that nanofluids have a significant potential for hydrodynamic flow and heat transfer enhancement and are highly appropriate for industrial and practical applications.

KEYWORDS: Forced Convection; Turbulent Flow; Nanofluids; CFD

INTRODUCTION

Performance of heat transfer equipment can be improved with studies related to a significant increase in heat flux and miniaturization. In many industrial applications such as power generation, microelectronics, heating processes, cooling processes and chemical

processes; water, mineral oil and ethylene glycol are used as heat transfer fluid. The effectiveness of heat exchange processes are obstructed by the lower heat transfer properties of these common fluids as compared to most solids. It is obvious that solid particles having thermal conductivities several hundred times higher than conventional fluids used in the heat transfer applications. To improve the thermal conductivity of a fluid; suspension of ultrafine solid particles in the fluid was used. Choi, [1] at Argonne National Laboratory proposed the use of nanoparticles to enhance the thermal conductivity of liquids, and coined the term 'nanofluids' for the resulting new class of fluids engineered by dispersing nanometer-sized materials in base fluids. Compared with the existing techniques for enhancing heat transfer, the nanofluids show a great potential in increasing heat transfer rates in a variety of application cases, with incurring either little or no penalty in pressure drop. The theoretical and experimental investigations on convective heat transfer in confined flows applications with nanofluids under turbulent conditions is an active area of research.

Bajestan et al. [2] investigated a novel method of improving the efficiency of solar systems heat exchangers. For this purpose, they examined convective heat transfer of a TiO₂/water nanofluid flowing through a uniformly heated circular tube both experimentally and numerically. The study concluded that the heat transfer coefficient of the nanofluid is higher than that of the base fluid. Moreover, the Reynolds number and particle concentration, as well as nanoparticle thermal conductivity, caused the convective heat transfer coefficient of nanofluid to increase. Tiwari et al. [3], Hojjat et al. [4], Kim et al. [5], Duangthongsuk and Wongwises [6] and Xuan and Li [7], experimentally investigated the forced convective heat transfer through a circular straight tube with a constant temperature and heat flux condition in the laminar and turbulent flow regimes using nanofluids. The nanoparticles dispersed were Al₂O₃, CuO, and TiO₂ nanoparticles in an aqueous solution of carboxymethyl cellulose (CMC) and water. Their results showed that the local and average heat transfer coefficients of nanofluids were larger than those of the base fluid. Also, they found that thermal conductivity and convective heat transfer coefficient were increased up to 8% and 20% respectively, in alumina nanofluids for three different volume concentrations of suspended particles.

Moreover, the number of numerical studies by implementing CFD programs on nanofluids is growing and the analysis can be carried out by using two approaches that are, one phase and two phase models. The combined effect of using helical coils and nanofluids heat transfer enhancement and pressure losses in turbulent flow was numerically investigated by Elsyed et al. [8]. The developed CFD models were validated against experimental data and empirical correlations. Results showed that using 3% volume fraction of Al₂O₃/water nanofluids in helical coils increased the heat transfer coefficient by up to 60% of that for pure water in straight tubes at the same Reynolds number. While the turbulent forced convection flow of water-Al₂O₃ nanofluid in a circular tube was investigated numerically with constant and uniform heat flux at the wall by Bianco et al. [9]. The study was based on two different approaches: single and twophase models. It was calculated that convective heat transfer coefficient for nanofluids was greater than that of the base liquid. Also, their study showed that the heat transfer enhancement is increased with the particle volume concentration and Reynolds number. Guzei et al. [10], Hatami and Okhovati [11], Rostamani et al. [12], Namburu et al. [13] and Ali et al. [14] numerically investigated the turbulent flow of nanofluids with different volume concentrations of nanoparticles (CuO, Al₂O₃, TiO₂ and SiO₂) in an ethylene glycol and water mixture flowing through a circular tube under constant heat flux

condition. It was found that when the volume concentration is increased the wall shear stress and heat transfer rates augmented. They compared the convective heat transfer coefficients with each other and found that Nusselt number improved by 35% for 6% CuO nanofluids over the base fluid at a constant Reynolds number. Therefore, theoretical and experimental research works is needed to clearly understand and accurately predict the hydrodynamic and thermal characteristics of nanofluids.

A common ground in previous nanofluid studies is that all nanofluids used were made to be homogeneous and uniform suspension of nanoparticles has been shown to enhance heat transfer. The enhancement comes at a significant cost in pumping power consumption due to significant increases in nanofluid density and viscosity with increasing nanoparticles concentration. Hence, the purpose of this paper to numerically investigate the effect of different nanofluid parameters namely; base fluid type, nanoparticle material and nanoparticles concentration on heat transfer enhancement and pumping power consumption under turbulent pipe flow conditions.

PROBLEM FORMULATION

The governing equations

A two-dimensional, steady, incompressible, temperature-constant properties turbulent flow conditions are considered in this study. The compression work and viscous dissipation are negligible. A single-phase model was adopted because nanofluids may be considered as Newtonian fluids for low volume concentration fractions. For turbulent flow, most commercial software uses the Reynolds Averaged Navier-Stokes equations (RANS) model. It takes less computational effort than the time accurate equations and is robust for a wide range of fluid flows. It is derived from the standard equations by averaging after decomposing the flow variables into mean and fluctuating components like $\chi = \bar{\chi} + \chi'$, where $\bar{\chi}$ is the mean and χ' is the fluctuating component of variables like velocity, pressure or other scalar quantities. This formulation leads to the continuity and momentum equations coupled with the standard $k - \varepsilon$ closer turbulent model. According to Fluent theory guide [15] the governing equations are:

The continuity equation:

$$\frac{\partial}{\partial x_i} (\rho u_i) = 0 \tag{1}$$

RANS equation:

$$\frac{\partial}{\partial x_j} \left(\rho u_j u_i \right) = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left[\mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_l}{\partial x_l} \right) \right] + \frac{\partial}{\partial x_j} \left(-\rho \overline{u'_i u'_j} \right)$$
(2)

Energy equation

$$\frac{\partial}{\partial x_j} \left[u_i \left(\rho E + P \right) \right] = \frac{\partial}{\partial x_j} \left[\left(k + \frac{c_p \mu_t}{P r_t} \right) \frac{\partial T}{\partial x_i} + u_i \left(-\rho \overline{u'_i u'_j} \right) \right]$$
(3)

A constant value for the turbulent Prandtl number, $Pr_t = 0.85$, was used in all computations. The apparent turbulent shearing stresses might be related to the rate of mean strain through an apparent scalar turbulent or "eddy" viscosity.

For the general Reynolds stress tensor, the Boussinesq assumption gives:

$$-\rho \overline{u_i' u_j'} = \mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \left(\rho k + \mu_t \frac{\partial u_l}{\partial x_l} \right) \delta_{ij}$$
(4)

where δ_{ij} is the Kronecker delta function ($\delta_{ij} = 1$ if i = j and $\delta_{ij} = 0$ if $i \neq j$), k is the turbulent kinetic energy and μ_t is the turbulent viscosity.

The k-equation

$$\frac{\partial}{\partial x_j} \left(\rho u_j k \right) = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + 2 \mu_t S_{ij} S_{ij} - \beta_1 \rho \varepsilon$$
(5)

The ε-equation

$$\frac{\partial}{\partial x_j} \left(\rho u_j \varepsilon \right) = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_{\varepsilon}} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + 2 C_{1\varepsilon} \frac{\varepsilon}{k} \mu_t S_{ij} S_{ij} - C_{2\varepsilon} \frac{\varepsilon^2}{k}$$
(6)

where:

$$\mu_t = \rho C_\mu \, k^2 / \varepsilon$$

The standard values of coefficients for $(k-\epsilon)$ equations are: $C_{\mu} = 0.09$; $\sigma_k = 1.0$; $\sigma_{\epsilon} = 1.3$; $C_{\epsilon 1} = 1.44$; $C_{\epsilon 2} = 1.92$; $\beta_1 = 1.0$.

Nanofluid thermo physical properties

The presence of nanoparticles influences the working fluid properties. The four main properties involved in calculating heat transfer rate of the nanofluid are density, specific heat, viscosity, and thermal conductivity, which may be quite different from those of the original pure fluid. The density and specific heat were calculated by assuming the thermal equilibrium between particles and surrounding fluid [16-17].

$$\rho_{nf} = \phi \ \rho_p + (1 - \phi) \rho_f \tag{7}$$

$$\left(c_{p}\right)_{nf} = \frac{\phi \left(\rho c_{p}\right)_{p} + (1-\phi) \left(\rho c_{p}\right)_{f}}{\phi \rho_{p} + (1-\phi) \rho_{f}}$$

$$\tag{8}$$

The classical Maxwell model for effective thermal conductivity for solid-liquid mixtures was used [18].

$$\frac{k_{nf}}{k_f} = \frac{k_p + 2k_f + 2\phi k_f (k_p - k_f)}{k_p + 2k_f - \phi k_f (k_p - k_f)}$$
(9)

Batchelor modified Einstein relation to calculate the effective viscosity by considering the nanoparticle Brownian motion and their interaction [19].

$$\mu_{nf} = (1 + 2.5 \phi + 6.5 \phi^2) \mu_f \tag{10}$$

GEOMETRIC CONFIGURATION AND BOUNDARY CONDITIONS

In the numerical simulations, the heated tube having 0.05 m diameter and 2.0 m length was used. The flow is axisymmetric, so that only half of its domain is used in the computation. The computational domain and the boundary conditions are presented in Figure (1).

A uniform flow velocity and a constant temperature were assigned at the inlet section. The Reynolds number was varied in the range of 5000 - 30,000. No slip boundary condition at the wall where both velocity components were set to zero while the heating

conditions were constant wall temperature of 333 K or uniform heat flux of 50000 W/m^2 . Pressure outlet conditions are applied to define the static pressure at flow outlet.



Figure 1: Geometric domain and boundary conditions.

NUMERICAL GRID AND PROCEDURE

The system of governing equations was solved by implementing the commercial powerful code ANSYS Fluent R14.5. Structured mesh method done in ANSYS Workbench. The mesh was non-uniform in order to optimize distribution in the entrance and wall regions. The meshing is done with inflation control is set to "Program Controlled" and the named selections are defined and the meshing is generated.

All governing equations are discretized by a second order scheme and then solved, the flow field is assumed to be incompressible and the SIMPLE (Semi-Implicit Method for Pressure-Linked Equations) algorithm is used to couple the velocity and pressure fields.

The convergence criterion for all the dependent variables is specified as 10^{-6} . The default values of under-relaxation factor for pressure, density, body force, momentum, turbulent kinetic energy, turbulent dissipation rate, turbulent viscosity and energy are 0.3, 1.0, 1.0, 0.7, 0.8, 0.8, 1.0 and 1.0 respectively used in the simulation work. Solution initialization is done. Then the number of iterations is defined as 1500 and the calculations proceed. The iterations continue until the convergence is reached.

VERIFICATION AND VALIDATION ANALYSIS

There are mainly two sources of uncertainty in CFD, namely modeling and numerical [20]. Modeling uncertainty can be evaluated through results validation while numerical uncertainty can be approximated through grid independence. Numerical uncertainty has two main sources, namely truncation and round-off errors. Higher order schemes have less truncation error; therefore the discretization schemes invoked were second-order. In explicit schemes, round-off error increases with increasing iterations. However, having used Gauss-Seidel iterative procedure in a steady-state simulation renders the calculation insensitive to round-off error. The grid independence is also verified by calculation of the total heat flux at Re = 5000 for pure water. The results of different grids are shown in Figure (2). The grid (45 X 900 = 40500 cells) has been adopted because it ensured a good compromise between the machine computational time and the accuracy requirements. Therefore, both the numerical and modeling errors are considered negligible. In addition, the convergence is achieved at all testing calculations, a case of residuals history is shown in Figure (3).



Figure 2: Grid independence testing.



Figure 3: Convergence history for water/Al₂O₃ and $\varphi = 9\%$ nanofluids at Re = 20000.

Results are verified by comparing the obtained numerical results with the results obtained by the known Dittus–Boelter equation [21]. Comparisons in terms of average Nusselt number values are reported in Figure (4) for pure water in the tested turbulent range. It is observed that the present results fit very well with the equation.



Figure 4: Verification of numerical results.

RESULTS AND DISCUSSION

A computational analysis of turbulent flow of nanofluids with different volume concentrations of nanoparticles being copper (Cu), alumina (Al_2O_3) and titania (TiO_2) suspended in water, ethylene glycol and engine oil. The flow along the two-dimensional duct is under constant wall temperature and uniform heat flux condition. The results of the study are presented in terms of average and local convective heat transfer coefficients, Nusselt number, required pumping power profiles as a function of Reynolds number.

Effect of nanoparticles volume concentration

Figures (5) and (6) describes the local and average convective heat transfer coefficient distributions along the constant wall temperature surface at Re = 20000, for φ = 0%, 3%, 6%, and 9% water/Al₂O₃ nanofluids. Results reveal that the convective heat transfer coefficient of nanofluid increases with increase in volume concentrations of nanoparticles. This increase in heat transfer rate is mainly contributed to the increase in thermal conductivity of nanofluid by addition of nanoparticles and increased turbulence in the flow. Higher the concentration of nanoparticles in the base fluid increases its thermal conductivity and hence less resistance for heat transfer.

Uniform heat flux results are presented in terms of surface wall temperature as shown in Figure (7) for turbulent flow of water/ Al_2O_3 nanofluids at different concentrations. The results showed an enhancement on heat transfer and cooling process as the concentration increases.



Figure 5: Local heat transfer coefficients for water/Al₂O₃ nanofluids at Re = 20000 for constant wall temperature conditions.



Figure 6: Average heat transfer coefficients for water/Al₂O₃ nanofluids.



Figure 7: Wall temperature for water/Al₂O₃ nanofluids at Re = 20000 for uniform heat flux conditions.

Effect of nanoparticles materials

The local heat transfer coefficient and the Nu number results are plotted in Figures (8) and (9) respectively, for constant wall temperature, the results are for water and different nanoparticles materials at volume fraction $\varphi = 9\%$. It is assumed that the size and the shape of the nanoparticles is the same during all calculations. It can be seen that the heat transfer enhancement increases with the presence of nanoparticles and this increase depends on nanoparticle material thermal conductivity, this enhancement is more for metals, which is Cu particles than metal oxides; Al₂O₃ and TiO₂.



Figure 8: Local heat transfer coefficients for water nanofluids and $\varphi = 9\%$ at Re = 20000 for constant wall temperature conditions.



Figure 9: Average Nu number for water nanofluids with different nanoparticles materials and $\varphi = 9\%$ for constant wall temperature conditions.

Also, for uniform heat flux, the results are presented in Figure (10) and showed that Cu particles nanoparticles enhancement of heat transfer is more than the other tested materials.



Figure 10: Wall temperature for water Nano fluids and $\varphi = 9\%$ with different nanoparticles materials.

Effect of base fluid

The results show increased thermal conductivity enhancement for poorer (lower thermal conductivity) heat transfer fluid. The results of forced convection process of nanofluids with volume fraction $\varphi = 9\%$, for different base fluids are shown in Figures (11) and (12) for constant wall and uniform heat flux conditions respectively. It can be observed that the least enhancement for water which is the best heat transfer fluid with the highest thermal conductivity of the compared fluids. This can be explained as the

nanoparticles and high-Prandtl number base fluid could significantly increase the heat transfer performance for nanofluids.



Figure 11: Local heat transfer coefficients for different base fluids with Al_2O_3 nanofluids at Re = 20000 for constant wall temperature conditions.



Figure 12: Wall temperature for different base fluids with Al₂O₃ nanofluids at Re=20000 for uniform heat flux conditions.

Pumping power requirements

The numerical calculations of the average friction factor results are used in calculation of pressure drop. Once the pressure drop (or head loss) is known, the required pumping power to overcome the pressure loss is determined by the relation $PP = \dot{V} \Delta P$. The pumping power ratio referred to the base fluid values are described in Figure (13) to water/AL₂O₃ nanofluids versus Reynolds number at different volume concentrations. It is observed that the ratio (PP_{nf}/PP_f) profiles tend to increase as the volume concentration

grows; however, it is observed that the ratio does not seem to be dependent on Re. The results indicate that the pumping power ratio is equal to 1.07, 1.17, and 1.30 for $\varphi = 3\%$, 6%, and 9%, respectively.



Figure 13: Pumping power profiles, referred to the base fluid for water/Al₂O₃ nanofluids.

CONCLUSIONS

The effect of different parameters on heat transfer enhancement by nanofluid is investigated numerically by implementing the ANSYS Fluent R14.5 code under turbulent flow conditions. Based on the numerical study of the problem the following conclusions can be drawn:

- Increaseasing particle volume fraction improves the heat transfer rates in nanofluids compared to those in the base fluid alone.
- The higher thermal conductivity of the base fluid the lower heat transfer enhancement resulted.
- The nanoparticle material has the least effect on heat transfer enhancement by nanofluids and it was noticed that the enhancement is more for metals than metal oxides.
- For nanofluids, increasing the inlet Reynolds number will enhance the convective heat transfer performance with limited penalties in increased pumping power requirement.

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NOMENCLATURE

- C_p Specific Heat (kJ/kg. K)
- D Diameter (m)
- h Heat transfer coefficient [w $(m^{-2} K^{-1})$]
- k Thermal conductivity $[w (m^{-1} K^{-1})]$
- L Length (m)
- Nu Nusselt number
- P Pressure [Pa]
- PP Pumping power (W)
- Pr Prandtl number
- Q Heat flux [W m⁻²]
- Re Reynolds number
- T Temperature [K]
- u Component of velocity [m s⁻¹]
- V Volume flow rate (m^3/s)

Greek symbols

- φ Volume fraction of nanoparticle
- ρ Density [kg/m³]
- μ Dynamic viscosity (Pa s)

Subscripts

- ave Average
- f Base fluid
- t Turbulent
- nf Nanofluid
- p Nanoparticle
- x Local