

A SOLVER-TOPSIS TECHNIQUE FOR MULTI-OBJECTIVE OPTIMIZATION OF INNOVATIVE MULTI-STAGE VCR SYSTEMS BY USING MICROSOFT EXCEL

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المخلص

تصف هذه الورقة طريقة لإجراء تحليلات تحسين الأداء متعدد الأهداف لأنظمة التبريد بضغط البخار متعدد المراحل باستخدام برنامج حل الهدف الواحد Microsoft Excel Solver مع طريقة اتخاذ القرار TOPSIS. توضح الورقة كيفية تطبيق الطريقة المقترحة على نظام مبتكر ذي مرحلتين يضيف إلى النظام التقليدي مبردًا داخلياً لاستعادة الحرارة بعد الضاغط الأول لإنتاج ماء ساخن. يقوم النموذج الحاسوبي للنظام الذي تم تطويره باستخدام Excel بحساب معامل الأداء، الكفاءة الأوكسيريكية، معدل التكلفة الإجمالي، وإجمالي تأثير الاحتراز المكافئ باعتبار أربعة خصائص للتصميم كمتغيرات وهي درجة حرارة المبخر والمكثف، كفاءة الضاغط الحرارية، ومعدل سريان الماء في المبرد. بمقارنة مؤشرات الأداء الأربعة للنظام التي تم تحسينها بالطريقة المقترحة مع تلك المحسنة باستخدام برنامج MIDACO متعدد الأهداف يتضح أن الطريقة المقترحة تفرز قيماً أعلى لمعامل الأداء والكفاءة الأوكسيريكية مع خفض في إجمالي تأثير الاحتراز المكافئ على حساب زيادة طفيفة في معدل التكلفة الإجمالي للنظام. كما أن المعدل الكلي لتحطم الأوكسيريكي في النظام المحسن بهذه الطريقة أقل من ذلك المحسن ب MIDACO.

ABSTRACT

This paper describes a method for multi-objective optimisation analyses of multi-stage vapour-compression refrigeration (VCR) systems by utilising the single-objective solver of Microsoft Excel and the TOPSIS decision-making method. The paper illustrates the Solver-TOPSIS method by analysing an innovative two-stage VCR system that adds a heat-recovery intercooler to the conventional system after the first-stage compressor. The Excel-aided model developed for the system calculates its coefficient of performance (COP), exergetic efficiency, total cost rate, and total equivalent warming impact (TEWI). Four design parameters are treated as variables which are the evaporator and condenser temperatures, the isentropic efficiency of the compressor, and the flow rate of the cooling water. Comparing the values of the four performance indicators as optimised by the proposed method with those optimised by the MIDACO solver shows that the method yields higher values of the system's COP and exergetic efficiency and lower TEWI at the expense of slightly increasing the total cost rate. The total rate of exergy destruction in the system optimised by excel method is also lower than in the one optimised by MIDACO.

KEYWORDS: Multi-Stage VCR Systems; Multi-Objective Optimisation; TOPSIS; Excel.

INTRODUCTION

The significant share of vapour-compression refrigeration (VCR) systems in the energy consumption of residential, commercial, and industrial sectors emphasises the importance of improving the performance of these systems [1,2]. In this respect, multi-stage compression allows various innovative methods to be used for reducing the energy consumption of these systems. Since the improved systems cost more than the simple

ones, their economic feasibility requires careful balances between their electrical energy consumption and capital costs. A third factor has now become equally important due to the increasing concern about the harmful effects of global warming and ozone-layer depletion, which is the need to replace the usual synthetic refrigerants with more environment-friendly fluids [3]. The quest to design innovative VCR systems using environment-friendly refrigerants and to develop suitable methods for their energetic, economic, and environmental optimisation has inspired many researchers to be involved. Roy and Mandal [4] conducted a thermo-economic analysis of a simple VCR system using three refrigerants with low global-warming potential (GWP) namely, R152a, R600a and R1234ze. Developing their model with Engineering Equation Solver (EES), they evaluated the effect of evaporator and condenser temperatures on the system's coefficient of performance (COP), exergetic efficiency, and annual plant cost rate. They carried out a multi-objective optimisation (MOO) analysis by using MATLAB toolbox and used the TOPSIS method [5] to select the best optimised solution. Their results showed that R152a gave the best performance among the three investigated refrigerants.

Aminyavari et al. [6] analysed a 50-kW CO₂/NH₃ cascade refrigeration system by evaluating its exergetic, economic, and environmental performance. They also developed their model in MATLAB but employed a specific MATLAB function for calling the REFPROP data base [7] to obtain the refrigerants' thermodynamic data. Their MOO analyses used a genetic algorithm method to achieve the optimal design parameters of the system and used TOPSIS to select the final optimum point from the set of optimal solutions achieved. Their results showed that the optimum design results in exergetic efficiency of 45.89% and a total cost rate of 0.01099 US\$/s. Singh et al. [8] analysed an ammonia-based multi-stage compression VCR system incorporated with a flash intercooler which also works as a sub-cooler. They carried out a thermo-economic optimisation of the system in order to maximise its exergetic efficiency and minimise its total capital cost rate. The evaporator temperature, condenser temperature, subcooling parameter, and de-superheating parameter were considered as design variables for their MOO analysis. They employed the multi-objective genetic algorithm tool provided with MATLAB to carry out the optimisation analysis and used EES to determine the thermodynamic properties of the refrigerants. TOPSIS was used to select unique solutions for five different weighting factors of exergetic efficiency and total cost. Their results revealed that the exergetic efficiency and total capital cost of the system at the thermo-economic optimal operating conditions were 41.76% and 223,717.6 USD, respectively.

The above example studies show that most researchers used commercial software for their MOO analyses like MATLAB, EES, and REFPROP. However, the use of general-purpose applications can encourage independent researchers and engineering students to contribute to the development of innovative VCR systems using environment-friendly refrigerants. Microsoft Excel, which is an easy-to-learn general-purpose spreadsheet application, has a versatile solver for single-objective optimisation (SOO) analyses. Regarding MOO analyses, a free version of the MIDACO solver [9] is available for Excel users, but it allows only four design variables to be considered in the analysis; which is not adequate for analysing multi-stage compression and cascade VCR systems with multiple design parameters. Since the development of MOO solvers is too complex and takes a long time even for top software professionals [10], El-Awad [11] described a method for using the same SOO Solver provided by Excel for conducting MOO analyses with practically any number of design variables by utilising the TOPSIS method. The present paper applies the method for energetic, exergetic, economic, and environmental (4E) optimisation of an innovative two-stage compression VCR system.

Anjum et al. [12] conducted a thermodynamic analysis of the original two-stage system by using ammonia as the refrigerant and the input parameters shown on Table (1). The present analysis assumes the same input parameters, but deals with a multi-objective optimisation analysis of the improved system with R152a as refrigerant.

Table 1: Assumed values of the input parameters for analysing the VCR system [12]

Parameter	Value
Cooling capacity of the system, CC	10 kW
Evaporator temperature, T_E	-15°C
Condenser temperature, T_C	40°C
Ambient temperature, T_0	25°C
Temperature change for air in evaporator and condenser, ΔT	$\pm 5^\circ\text{C}$
Temperature of the inlet air to evaporator, T_{air}	0°C
Inlet temperature of cooling water, T_{w1}	17°C
Isentropic efficiency of compressor, η_c	80%
Overall heat transfer coefficient for evaporator, U_{eva}	0.03 kW/m ² .K
Overall heat transfer coefficient for condenser, U_{con}	0.04 kW/m ² .K
Overall heat transfer coefficient for intercooler, U_{intr}	0.1 kW/m ² .K
Maintenance factor, ϕ	1.06
Interest rate, i	14%
Plant life time, n	15 Years
Annual operation hours, N	4266 hour
Electrical power cost, c_{elec}	0.09 \$/kWh
Emission factor, μ_{CO_2e}	0.968 kg/kWh [13]
Cost of CO ₂ avoided, c_{CO_2}	0.09 \$/kg of emitted CO ₂

THE THERMODYNAMIC MODEL

Table (2) shows the mass and energy balance equations for the different system components. The mass flow rate of the refrigerant in the evaporator is given by:

$$\dot{m}_r = CC / (h_1 - h_8) \quad (1)$$

where CC is the cooling capacity of the system. The temperatures T_2 and T_4 are determined from the temperatures T_{2s} and T_{4s} following isentropic compression processes:

$$T_2 = T_1 + (T_{2s} - T_1) / \eta_c \quad (2)$$

$$T_4 = T_3 + (T_{4s} - T_3) / \eta_c \quad (3)$$

The exit temperature of the cooling water T_{w2} is determined from the specified value of the heat-exchanger effectiveness, ε :

$$T_{w2} = T_{w1} + \varepsilon(T_2 - T_{w1}) \quad (4)$$

The enthalpy of the cooled refrigerant is then determined from energy balance across the intercooler as shown on Table (2). The total compression work is given by:

$$\dot{W}_{Total} = \dot{W}_{comp1} + \dot{W}_{comp2} = \dot{m}_r (h_2 - h_1) + \dot{m}_r (h_4 - h_3) / (1 - x_6) \quad (5)$$

The COP and overall exergetic efficiency, ε , of the system are given by:

Table 2: Mass and energy balance equations and exergy destruction rates in the system components

	Mass balance	Energy balance	Exergy destruction rate
Evaporator	$\dot{m}_1 = \dot{m}_8 = \dot{m}_r$	$\dot{m}_1 h_1 = \dot{m}_8 h_8 + CC$	$T_0 \left[\dot{m}_1 (s_1 - s_8) - \frac{CC}{T_E} \right]$
Compressor 1	$\dot{m}_2 = \dot{m}_1$	$\dot{W}_{comp1} = \dot{m}_r (h_2 - h_1)$	$T_0 \dot{m}_1 (s_2 - s_1)$
Compressor 2	$\dot{m}_4 = \dot{m}_9 = \dot{m}_r / (1 - x_6)$	$\dot{W}_{comp2} = \dot{m}_3 (h_4 - h_9)$	$T_0 \dot{m}_3 (s_4 - s_9)$
Condenser	$\dot{m}_5 = \dot{m}_4 = \dot{m}_r / (1 - x_6)$	$\dot{m}_5 h_5 = \dot{m}_4 h_4 - \dot{Q}_{cond}$	$T_0 \left[\dot{m}_4 (s_5 - s_4) + \frac{\dot{Q}_{cond}}{T_C} \right]$
Throttle valve 1	$\dot{m}_6 = \dot{m}_5 = \dot{m}_r / (1 - x_6)$	$h_6 = h_5$	$\dot{m}_5 T_0 (s_6 - s_5)$
Throttle valve 2	$\dot{m}_8 = \dot{m}_7 = \dot{m}_r$	$h_8 = h_7$	$\dot{m}_7 T_0 (s_8 - s_7)$
Flash chamber	$\dot{m}_3 = x_6 \dot{m}_5$ $\dot{m}_7 = (1 - x_6) \dot{m}_5 = \dot{m}_r$	$h_3 = h_g @ P_{fc}$ $h_7 = h_f @ P_{fc}$ $\dot{m}_6 h_6 = \dot{m}_3 h_8 + \dot{m}_7 h_7$	$\frac{T_0 \dot{m}_1}{1 - x_6} \times [s_6 - x_6 s_3 - (1 - x_6) s_7]$
Intercooler	$\dot{m}_2 = \dot{m}_r$ $\dot{m}_{w1} = \dot{m}_{w2}$	$h_{2b} = h_2 - \dot{m}_w c_{pw} (T_{w2} - T_{w1}) / \dot{m}_2$	$T_0 [\dot{m}_w s_{w2} + \dot{m}_r s_{2b} - \dot{m}_w s_{w1} - \dot{m}_r s_2]$
Direct heat exchanger	$\dot{m}_9 = \dot{m}_2 + \dot{m}_3$	$\dot{m}_2 h_{2b} + \dot{m}_3 h_3 = \dot{m}_9 h_9$	$T_0 [\dot{m}_9 s_9 - \dot{m}_r s_{2b} - \dot{m}_3 s_3]$

$$COP = \frac{CC}{\dot{W}_{Total}} \quad (6)$$

$$\varepsilon = \frac{CC \times [(T_C + 273.15) / (T_E + 273.15) - 1]}{\dot{W}_{Total}} \quad (7)$$

The total exergy destruction in the system \dot{E}_{Total}^D is given by:

$$\dot{E}_{Total}^D = \dot{E}_{Evap}^D + \dot{E}_{Comp1}^D + \dot{E}_{TV1}^D + \dot{E}_{IC}^D + \dot{E}_{FC}^D + \dot{E}_{DCHX}^D + \dot{E}_{Comp2}^D + \dot{E}_{TV2}^D + \dot{E}_{Cond}^D \quad (8)$$

The exergy destruction rates in the different system components are shown on Table (2).

THE ECONOMIC MODEL

The total annualised cost rate of the system is given by [12]:

$$\dot{C}_{total} = \sum_{k=1}^9 \dot{C}_k + \dot{C}_{op} + \dot{C}_{env} \quad (9)$$

where, \dot{C}_k is the capital and maintenance cost rate of individual components, \dot{C}_{op} is the operation cost rate of the system, and \dot{C}_{env} is the CO₂ penalty cost rate of the system. The total capital and maintenance cost rate of the system is calculated by adding up the capital and maintenance cost rate of the individual components which is given by:

$$\dot{C}_k = C_k \cdot \phi \cdot CRF \quad (10)$$

where, C_k is the capital cost of the component, ϕ is the maintenance factor, and CRF is the capital recovery factor obtained from:

$$CRF = \frac{i(1+i)^n}{(1+i)^n - 1} \quad (11)$$

where i is the interest rate and n is the system's expected lifetime. The values of i and n used in the present analysis are given in Table (1).

The costs of the two throttle valves, flash chamber, and direct contact heat exchanger have been ignored in some analyses because they are minor compared to those of the two compressors, the evaporator, and the condenser. In the present analysis, the cost of the DCHX is taken as equal to that of the flash chamber. The capital costs of individual components are estimated by using the relations shown on Table (3).

Table 3: Capital cost functions of the different components [8, 12]

Component	Capital cost function
Evaporator	$C_{eva} = 1397 \times A_{eva}^{0.89}$
Compressor	$C_{comp} = 10167.5 \times \dot{W}^{0.46}$
Condenser	$C_{con} = 1397 \times A_{con}^{0.89}$
Throttle valve	$C_{TV} = 114.5 \times \dot{m}$
Flash chamber	$C_{FC} = 280.3 \dot{m}_{ref}^{0.67}$
Intercooler	$C_{intr} = 2382.9 \times (A_{intr})^{0.68}$

The heat-transfer areas of the evaporator, condenser, and intercooler in the relations shown on Table (3) are determined by using the log-mean temperature difference method. The operational cost rate of the system is the cost of electricity given by:

$$\dot{C}_{op} = \dot{W} \cdot N \cdot c_{elec} \quad (12)$$

where N is the annual operational hours and c_{elec} is the cost of electricity in \$/kWh. Following Wang et al. [14], the penalty for the system's CO₂ emission is calculated from:

$$\dot{C}_{env} = m_{CO_2e} \cdot c_{CO_2} \quad (13)$$

where, c_{CO_2} is the penalty cost of the avoided CO₂ emission and m_{CO_2e} is the amount of annual CO₂ emission from the system that can be estimated from:

$$m_{CO_2e} = \mu_{CO_2e} \cdot E_{annual} \quad (14)$$

where μ_{CO_2e} is the emission factor and E_{annual} is the annual amount of energy consumed by the system. The values of N , μ_{CO_2e} , c_{elec} , and c_{CO_2} are given in Table (1).

THE TOTAL EQUIVALENT WARMING IMPACT (TEWI)

TEWI is a non-monetary measure of the direct and indirect global warming effects of the refrigeration systems. The direct effect results from the refrigerants being directly released or leaked into the atmosphere and the indirect effect is caused by the CO₂ emissions in thermal power plants that use fossil fuels to produce the electrical energy. The refrigerant TEWI is calculated by using the following correlation [15].

$$TEWI = GWP_{ref} \left[m_{ref} \times L_{annual} \times n + m_{ref} \times (1 - \alpha) \right] + (E_{annual} \times \beta \times n) \quad (15)$$

where GWP_{Ref} is the GWP of the refrigerant, m_{ref} is the total refrigerant charge, L_{annual} is the refrigerant leakage rate, α is the recycling factor, E_{annual} is the energy consumed per year, and β is the electricity regional conversion factor. Table (4) shows how m_{ref} and L_{annual} are calculated and gives the values of α , β , and GWP_{ref} for R152a [15]. The underlined term on the right side of Equation (15) is the indirect part of the TEWI.

Table 4: TEWI analysis assumptions [15]

Parameter	m_{ref} [kg]	L_{annual} [%]	α	β [kg.CO ₂ /kWh]	GWP_{ref}
Assumed value	$\dot{m}_{ref}(240s)$	12.5	0.7	0.65	140

THE EXCEL-AIDED MODEL AND SOO ANALYSES BY USING SOLVER

Figure (3) shows the first sheet of the Excel-aided model developed for the VCR system. The first column on the left side of the sheet stores the assumed data for the thermodynamic model such as the evaporator and condenser temperatures, the mass flow rate and inlet temperature of the cooling water, the intercooler's effectiveness, etc. The following two columns determine the temperature, enthalpy, and entropy values of the refrigerant and water at the various states by using Thermo property functions [16]. The formula bar reveals the formula in cell **N13** that calculates ϵ according to Equation (7). The fourth column from the left calculates the rates of exergy destruction in all the system components. The last column on the right determines the mass flow rate of the refrigerant, the compression work and four performance indicators; COP, ϵ , C_{Total} , and TEWI.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	System 2														
2	Fluid	R152a													
3	T_E	-20	oC	h_1	492.94		h_9	520.80011		T_0	298.15	K	m_r	0.03548	kg/s
4	T_C	40	oC	s_1	2.1627		T_9	28.138245		P_0	101.325	kPa	w_c1	1.43947	kW
5				S_2s	2.1627		s_9	2.1468937		h_0	534.9557		w_c2	1.94103	kW
6	P_E	120.680	kPa	h_2s	525.39655		s_4s	2.1468937		s_0	2.3363573		W_tot	3.38050	kW
7	P_C	909.270	kPa	h_2	533.51069		h_4s	555.78642					Q_cond	13.01253	kW
8				T_2	40.474495		h_4	564.533		ED_evap	0.000374				
9	P_ic	331.256	kPa	Tw_2	34.605872		s_2	2.1900162		ED_comp1	0.2890		TEWI_D	2592.92	
10	T_ic	6.478	oC	h_2b	523.13986		s_3	2.1140823		ED_comp2	0.3431		TEWI_ID	209395.73	
11				T_2b	30.409084		s_4	2.1728184		ED_cond	0.0478				
12	eta_c	0.8		h_5	271.35		s_5	1.2411		ED_tvalv1	0.1891		COP	2.958146	
13				h_6	271.35		s_6	1.2553899		ED_tvalv2	0.0994		epsilon	70.112098	%
14	E_HX	0.75		x_6	0.2005944		s_7	1.0399188		ED_FC	2.069E-15		C_total	12336.717	\$
15	Tw_1	17	oC	h_7	211.09539		x_8	0.1362408		ED_DCHX	0.001633		TEWI	211988.65	
16	m_w	0.005	kg/s	h_8	211.09539		s_8	1.0493144		ED_IC	-0.0034				
17	CC	10	kW	h_3	511.4758		s_2b	2.1549728							
18															

Figure 3: Sheet 1 of the model for the two-stage compression VCR system

Figure (4) shows the second sheet of the model that calculates the cost rates of the nine system components using the relevant cost relations shown on Table (3). The formula bar shows the formula in cell **E12** that calculates the capital cost of the intercooler. Figure (4) shows that the total cost of the system is dominated by the costs of the evaporator, the condenser, and the two compressors. Solver, the single-objective solver that comes with Excel, can be used to optimise any of the four key performance indicators shown on Figure (3) by using either the GRG Nonlinear method or the Evolutionary method. Figure (5) shows the set-up for maximising the system's COP by using the Evolutionary method.

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2	SV	0		T_E	-20.000	oC	T_C	40.000	oC			
3	n	15										
4	i	0.14		PEC_com1	12022.30		C_comp1	1957.33741		Z_comp1	0.48635	
5	PWF	6.142168		PEC_com2	13794.58		C_comp2	2245.88159		Z_comp2	0.55805	
6	CRF	0.162809		PEC_evap	19360.10		C_evap	3151.99766		Z_evap	0.78320	
7	ϕ	1.06		PEC_con	25712.27		C_con	4186.18731		Z_con	1.04017	
8	Hours	4266		PEC_tval1	3.25		C_tval1	0.5287389		Z_tval1	0.00013	
9	U_eva	0.03		PEC_tval2	4.06		C_tval2	0.66141502		Z_tval2	0.00016	
10	U_cond	0.04		PEC_flsh	11.55		C_flsh	1.88121154		Z_flsh	0.00047	
11	U_intr	0.1		PEC_DCHX	11.55		C_DCHX	1.88121154		Z_DCHX	0.00047	
12	Tairin_eve	0	oC	PEC_intr	485.83		C_intr	79.0966974		Z_intr	0.01965	
13	Tairin_con	25	oC	Evaporator		Condenser		Intercooler				
14	ΔT	5	oC	ΔT_1	20.000	ΔT_1	15.000	ΔT_1	5.869	C_equip_an	9782.435	\$/y
15	Eleccost	0.09	\$/kWh	ΔT_2	15.000	ΔT_2	10.000	ΔT_2	13.409	C_elec_an	1297.907	\$/y
16	μ_{CO2e}	0.968	kg/kWh	LMTD_E	17.380	LMTD_C	12.332	LMTD_intr	9.125	C_CO2e_an	1256.374	\$/y
17	c_CO2e	0.09	\$/kg	A_ev	19.179	A_con	26.381	A_intr	0.096	C_total_an	12336.717	\$/y
18												

Figure 4: Sheet 2 of the model for the two-stage VCR system

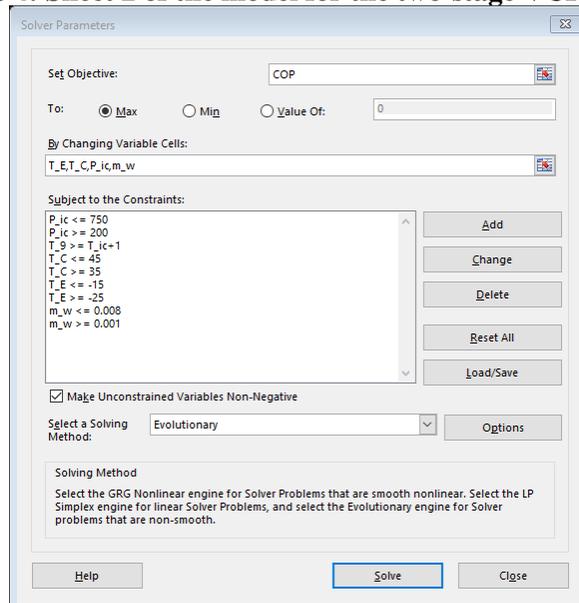


Figure 5: Solver set-up for maximising the COP of the VCR system

Figure (5) shows that the optimisation analysis involves four changing variables which are the evaporator temperature, the condenser temperature, the inter-stage pressure, and the flow rate of the cooling water. Figure (5) also shows the specified ranges within which the four variables are allowed to vary. The other three performance indicators were similarly optimised by selecting the respective objective cell and minimising or maximising its value with Solver. Table (5) shows the values of the four key-performance indicators for the base design and those obtained by the four optimised solutions.

Table 5: The single-objective optimised solutions obtained by Solver for the system

Optimisation objective	COP	ε [%]	C_{total} [\$/y]	TEWI [kg CO ₂ /y]
Base design	2,958	70.112	12337.00	211988.7
Maximise COP	3.474	71.555	14456.96	181066.2
Maximise ε	3.445	72.142	14200.60	182500.8
Minimise total cost rate	2.296	64.777	11166.29	272165.0
Minimise TEWI	3.574	71.816	14214.03	176028.0

MULTI-OBJECTIVE OPTIMISATION OF THE SYSTEM BY USING MIDACO

Although four performance indicators were considered in the previous single-objective optimisation analyses of the system, the dominance of the indirect global-warming effect on the TEWI enables the multi-objective optimisation analysis to be conducted with either the COP or TEWI. Therefore, the multi-objective optimisation analysis presented below involves only three performance indicators which are maximising the system's exergetic efficiency, minimising its total cost rate, and minimising its TEWI. The best trade-off between these three conflicting objectives can be found by using a MOO solver such as the MIDACO solver [9]. Although the free version of MIDACO is limited to four changing variables to be considered in the analysis, it is adequate for the present analysis in which the four changing variables are the same as those used for Solver and shown on Figure (5). Figure (6) shows MIDACO's set-up for the analysis. As a multi-objective optimisation solver, MIDACO produces a Pareto front containing a set of un-dominated optimum solutions from which the best optimum solution is selected. Figure (7) shows the Pareto front obtained by MIDACO and Figure (8) shows the selected 3E optimal solution.

Figure (9) shows the percentage deviations of the four single-objective solutions obtained by Solver as shown on Table (5) from the 3E solution obtained by MIDACO. Compared to Solver's three solutions that minimised the system's TEWI, maximised its exergetic efficiency, or maximised its COP, the 3E solution reduced both the COP and exergetic efficiency of the system and increased the TEWI. The trade-off for degrading these three performance indicators is that the 3E solution reduced the system's total cost rate. Compared to Solver's solution that minimised the total cost rate, the 3E solution increased the total cost rate but considerably increased both the COP and exergetic efficiency and reduced the TEWI. It can be judged from the scale of deviations shown on Figure (9) that the closest single-objective solution to the 3E solution is the one that maximised the system's exergetic efficiency followed by the one that maximised its COP.

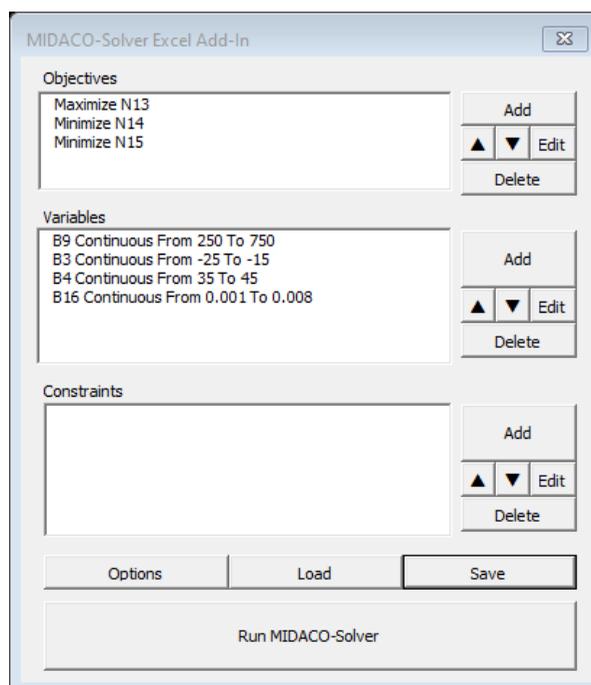


Figure 6: MIDACO's set-up for the 3E optimisation of the VCR system

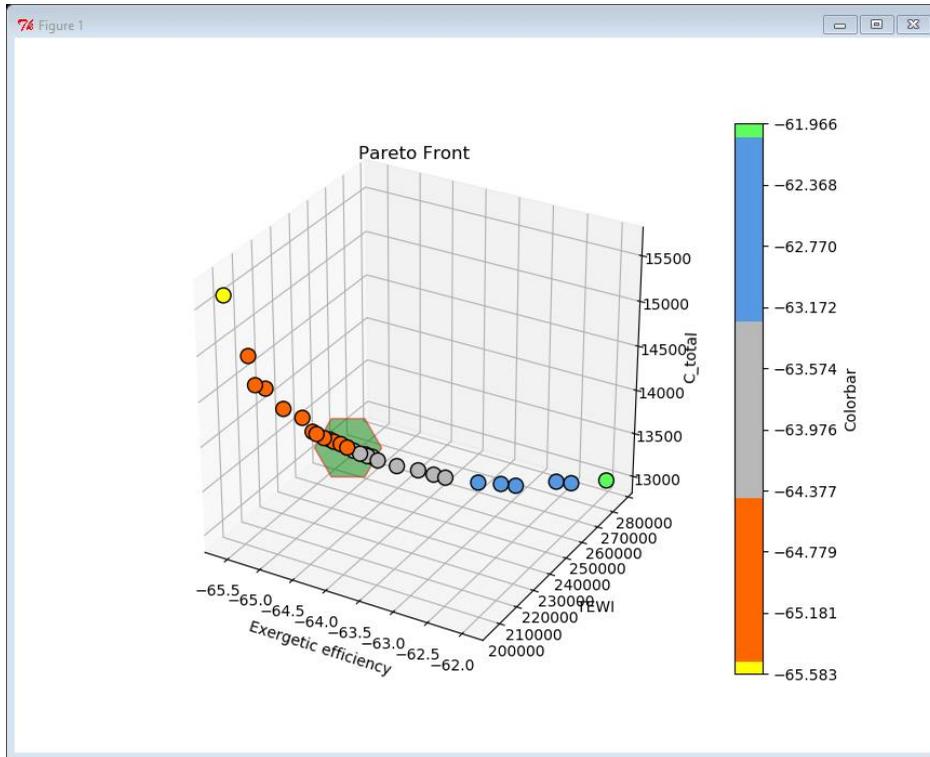


Figure 7: The Pareto front of the 3E optimised solution for the VCR system

=CC*((T_C+273.15)/(T_E+273.15)-1)/W_tot*100																
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O		
1	System 2															
2	Fluid	R152a														
3	T_E	-15	oC	h_1	496.57	h_9	525.91207	T_0	298.15	K	m_r	0.03540	kg/s			
4	T_C	41.43894	oC	s_1	2.15205	T_9	33.644462	P_0	101.325	kPa	w_c1	1.22515	kW			
5				S_2s	2.15205	s_9	2.1576934	h_0	534.9557		w_c2	1.93073	kW			
6	P_E	148.775	kPa	h_2s	524.25653	s_4s	2.1576934	s_0	2.3363573		W_tot	3.15589	kW			
7	P_C	944.898	kPa	h_2	531.17816	h_4s	560.77724				Q_cond	13.08779	kW			
8				T_2	38.721987	h_4	569.49353	ED_evap	0.000518							
9	P_ic	351.239	kPa	Tw_2	33.291491	s_2	2.1754399	ED_comp1	0.2469		TEWI_D	2587.09				
10	T_ic	8.213	oC	h_2b	529.25452	s_3	2.1114908	ED_comp2	0.3364		TEWI_ID	195483.025				
11				T_2b	36.867225	s_4	2.183163	ED_cond	0.0581							
12	η_c	0.8		h_5	274.06959	s_5	1.2495897	ED_tvalv1	0.1860		COP	3.169				
13				h_6	274.06959	s_6	1.2636722	ED_tvalv2	0.0765		ε	69.2763682	%			
14	ε_HX	0.75		x_6	0.2009176	s_7	1.0505007	ED_FC	0.000E+00		C_total	12425.490	\$			
15	Tw_1	17	oC	h_7	214.08994	x_8	0.1219283	ED_DCHX	0.003346		TEWI	198070.110				
16	m_w	0.001	kg/s	h_8	214.08994	s_8	1.0577531	ED_IC	0.0001							
17	CC	10	kW	h_3	512.61859	s_2b	2.1689933									

Figure 8: The selected 3E optimised solution obtained by MIDACO

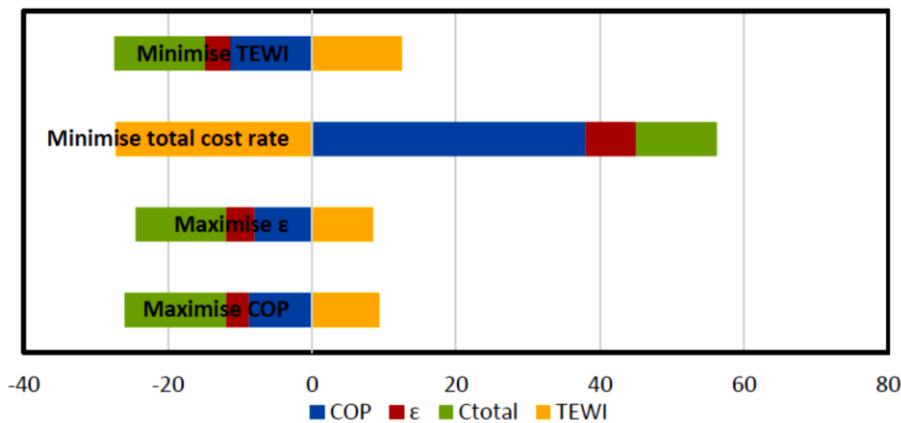


Figure 9: Percentage deviations of Solver's four single-objective solutions from the 3E optimised solution obtained by MIDACO

MOO ANALYSIS OF THE VCR SYSTEM BY USING THE SOLVER–TOPSIS TECHNIQUE

The TOPSIS decision-making technique ranks the different choices under consideration by evaluating an overall index, C_i , that measures the relative distances of these choices from the ideal choice according to the following relationship [6, 17]:

$$C_i = \frac{S_j^-}{S_j^+ + S_j^-} \tag{14}$$

Where S_j^+ and S_j^- are the distances from “benefit” and “non-benefit” ideal choices which are calculated based on the weighting factors provided to the method. Therefore, the method requires the benefit and non-benefit objectives as well as the weighting factors to be identified. This section shows how the single-objective solutions obtained by Solver can be used to achieve multi-objective optimisation of the VCR system by using the Solver-TOPSIS technique [11] that can be used to obtain a 3E optimised solution with any number of design variables and various weighting factors.

The Excel sheet shown on Figure (10) applies the TOPSIS method to the four single-objective optimised solutions obtained by Solver together with the 3E optimised solution obtained by MIDACO. The sheet is a modified version of an example sheet available at [18]. The values of the four performance indicators obtained by the five solutions are stored as a matrix in cells **B6:E10**. Note that there are two “benefit” objectives for this analysis, which are maximising the system’s COP and exergetic efficiency (ϵ), and two “non-benefit” objectives, which are minimising the system’s total cost rate (C_total) and TEWI. The sheet shown on Figure (10) applies a balanced weighting scheme that gives equal weights to these four objectives by assigning the value 0.25 to each of the four weighting factors W1 to W4 stored in cells **B4** to **E4**.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
3		W1	W2	W3	W4											
4	weightage	0.25	0.25	0.25	0.25	1										
5		COP	ε	C_total	TEWI			COP	ε	C_total	TEWI		Si+	Si-	Ci	Rank
6	Max COP	3.4739	71.555	14456.963	181066.2		Max COP	0.12	0.114	0.121	0.099		0.028	0.065	0.7	4
7	Max exg	3.4448	72.142	14200.6009	182500.8		Max exg	0.119	0.115	0.119	0.099		0.026	0.064	0.711	3
8	Min Ctotal	2.2963	64.78	11166.287	272165		Min Ctotal	0.08	0.104	0.093	0.148		0.07	0.028	0.284	5
9	Min TEWI	3.5742	71.82	14214.026	176028		Min TEWI	0.124	0.115	0.119	0.096		0.026	0.07	0.732	1
10	MIDACO	3.1687	69.28	12425.4904	198070.1		MIDACO	0.11	0.111	0.104	0.108		0.022	0.054	0.712	2
11																
12		COP	ε	C_total	TEWI											
13	Max COP	0.482	0.457	0.4841151	0.39476		V+	0.124	0.115	0.093	0.096					
14	Max exg	0.478	0.461	0.4755304	0.39789		V-	0.08	0.104	0.121	0.148					
15	Min Ctotal	0.318	0.414	0.3739214	0.59337											
16	Min TEWI	0.496	0.459	0.47598	0.38378											
17	MIDACO	0.439	0.443	0.4160879	0.43183											
18																

Figure 10: TOPSIS sheet for ranking the different optimised solutions obtained by Solver and MIDACO

As the formula bar shows, ranking of the five solutions is done by using Excel’s function “Rank”; which makes it easy to judge the different optimised solutions with different values of the four weight factors by giving more weight to any of these factors. With the balanced weighting scheme, Figure (10) shows that the solution that is nearest to satisfying the multi-objective requirement, i.e. the one with the largest value of C_i , is

not that obtained by MIDACO but that obtained by Solver for minimising the TEWI. The figure also shows that the solution with the smallest value of C_i is that for minimising the total cost rate.

By incorporating the TOPSIS sheet with the Excel-aided model of the VCR system, the scheme can be used not only to rank the four Solver solutions, but to maximise the value of C_i for the system by using Solver also. Figure (11) shows the third sheet that has been added to Excel-aided model shown on Figure (3). This sheet copies the values of the four performance indicators from Sheet 1 into its cells B9 to E9 as shown on Figure (11), while Sheet 1 copies the corresponding value of C_i from Sheet 3 into its cell N17 as shown on Figure (12). Note that the values of the four changing variables (the evaporator and condenser temperatures, the inter-stage pressure, and the water mass flow rate) are those of the base design that replaced the MIDACO solution in Sheet 3. Also note that Sheet 3 now shows the values of the four design variables, together with the value of T_{w2} , before Solver is used to maximise the value of C_i by adjusting the four changing variables in Sheet 1. Figures (13 and 14) show Sheet 1 and Sheet 3 with the solution obtained by Solver with the Evolutionary method and same constraints shown on Figure (5).

	B	C	D	E		H	I	J	K	L	M	N	O	P
	Benf. W1	Benf. W2	Non Benf. W3	Non Benf. W4		T_E	-20	oC	P_ic	331.3	kPa	m_w	0.005	kg/s
						T_C	40	oC				T_w2	34.61	oC
weightage	0.25	0.25	0.25	0.25	1									
	COP	ε	C_total	TEWI		COP	ε	C_total	TEWI		Si+	Si-	Ci	Rank
Max COP	3.4739	71.555	14456.963	181066.2		Max COP	0.122	0.114	0.121	0.097	0.028	0.065	0.699	3
Max exg	3.4448	72.142	14200.6009	182500.8		Max exg	0.121	0.115	0.119	0.098	0.026	0.064	0.71	2
Min Ctotal	2.2963	64.78	11166.287	272165		Min Ctotal	0.081	0.103	0.094	0.146	0.069	0.028	0.284	5
Min TEWI	3.5742	71.82	14214.026	176028		Min TEWI	0.125	0.114	0.119	0.095	0.026	0.069	0.731	1
Base design	2.9581	70.11	12336.7173	211988.6		Base design	0.104	0.112	0.103	0.114	0.031	0.044	0.591	4
	COP	ε	C_total	TEWI										
Max COP	0.488	0.456	0.4847129	0.38951		V+	0.125	0.115	0.094	0.095				
Max exg	0.484	0.46	0.4761176	0.3926		V-	0.081	0.103	0.121	0.146				
Min Ctotal	0.322	0.413	0.3743831	0.58548										
Min TEWI	0.502	0.458	0.4765677	0.37867										
Base design	0.415	0.447	0.4136253	0.45603										

Figure 11: Sheet 3 of the extended Excel-aided model for the system (base design)

	B	C	D	E	F	G	H	I	J	K	L	M	N	O
System 2														
Fluid	R152a													
T_E	-20	oC	h_1	492.94		h_9	520.80011		T_0	298.15	K	m_r	0.03548	kg/s
T_C	40	oC	s_1	2.1627		T_9	28.138245		P_0	101.325	kPa	w_c1	1.43947	kW
			s_2s	2.1627		s_9	2.1468937		h_0	534.9557		w_c2	1.94103	kW
P_E	120.680	kPa	h_2s	525.39655		s_4s	2.1468937		s_0	2.3363573		W_tot	3.38050	kW
P_C	909.270	kPa	h_2	533.51069		h_4s	555.78642					Q_cond	13.01253	kW
			T_2	40.474495		h_4	564.533		ED_evap	0.000374				
P_ic	331.256	kPa	Tw_2	34.605872		s_2	2.1900162		ED_comp1	0.2890		TEWI_D	2592.92	
T_ic	6.478	oC	h_2b	523.13986		s_3	2.1140823		ED_comp2	0.3431		TEWI_ID	209395.73	
			T_2b	30.409084		s_4	2.1728184		ED_cond	0.0478				
η_c	0.8		h_5	271.35		s_5	1.2411		ED_tvalv1	0.1891		COP	2.958146	
			h_6	271.35		s_6	1.2553899		ED_tvalv2	0.0994		ε	70.112098	%
ε_HX	0.75		x_6	0.2005944		s_7	1.0399188		ED_FC	2.069E-15		C_total	12336.717	\$
Tw_1	17	oC	h_7	211.09539		x_8	0.1362408		ED_DCHX	0.001633		TEWI	211988.65	
m_w	0.005	kg/s	h_8	211.09539		s_8	1.0493144		ED_IC	-0.0034				
CC	10	kW	h_3	511.4758		s_2b	2.1549728					TOPSIS Ci	0.5907068	

Figure 12: Sheet 1 of the extended Excel-aided model for the system (base design)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	System 2														
2	Fluid	R152a													
3	T_E	-15	oC	h_1	496.57		h_9	523.06305		T_0	298.15	K	m_r	0.03589	kg/s
4	T_C	38.01002	oC	s_1	2.15205		T_9	31.625241		P_0	101.325	kPa	w_c1	1.35216	kW
5				s_2s	2.15205		s_9	2.1396281		h_0	534.9557		w_c2	1.53161	kW
6	P_E	148.775	kPa	h_2s	526.71405		s_4s	2.1396281		s_0	2.3363573		W_tot	2.88377	kW
7	P_C	861.928	kPa	h_2	534.25007		h_4s	551.47574					Q_cond	12.54760	kW
8				T_2	42.319199		h_4	558.57891		ED_evap	0.000518				
9	P_ic	377.971	kPa	Tw_2	35.9894		s_2	2.1772295		ED_comp1	0.2694		TEWI_D	2622.49	
10	T_ic	10.410	oC	h_2b	524.88202		s_3	2.108305		ED_comp2	0.2750		TEWI_ID	178627.6	
11				T_2b	33.364045		s_4	2.1610152		ED_cond	0.0356				
12	η_c	0.8		h_5	267.6188		s_5	1.2292596		ED_tvalv1	0.1282		COP	3.467679	
13				h_6	267.6188		s_6	1.2392315		ED_tvalv2	0.0923		ε	71.207344	%
14	ε_HX	0.75		x_6	0.1678746		s_7	1.063903		ED_FC	0.000E+00		C_total	13526.753	\$
15	Tw_1	17	oC	h_7	217.90396		x_8	0.1337839		ED_DCHX	0.001199		TEWI	181250.09	
16	m_w	0.004235	kg/s	h_8	217.90396		s_8	1.0725282		ED_IC	-0.0011				
17	CC	10	kW	h_3	514.04671		s_2b	2.1458352					TOPSIS Ci	0.7654765	
18															

Figure 13: Sheet 1 of the extended Excel-aided model for the optimised system

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
2		Benf.	Benf.	Non Benf.	Non Benf.			T_E	-15	oC	P_ic	378	kPa	m_w	0.004	kg/s
3		W1	W2	W3	W4			T_C	38.01	oC				T_w2	35.99	oC
4	weightage	0.25	0.25	0.25	0.25	1										
5		COP	ε	C_total	TEWI			COP	ε	C_total	TEWI		Si+	Si-	Ci	Rank
6	Max COP	3.4739	71.555	14456.963	181066.2		Max COP	0.118	0.114	0.119	0.1		0.027	0.065	0.704	4
7	Max exg	3.4448	72.142	14200.6009	182500.8		Max exg	0.117	0.115	0.117	0.101		0.026	0.064	0.715	3
8	Min Ctotal	2.2963	64.78	11166.287	272165		Min Ctotal	0.078	0.103	0.092	0.151		0.07	0.027	0.28	5
9	Min TEWI	3.5742	71.82	14214.026	176028		Min TEWI	0.122	0.114	0.117	0.097		0.025	0.07	0.735	2
10	Base design	3.4677	71.21	13526.7533	181250.1		Base design	0.118	0.113	0.111	0.1		0.02	0.065	0.765	1
11																
12		COP	ε	C_total	TEWI											
13	Max COP	0.473	0.455	0.4765391	0.40088		V+	0.122	0.115	0.092	0.097					
14	Max exg	0.469	0.459	0.4680888	0.40406		V-	0.078	0.103	0.119	0.151					
15	Min Ctotal	0.312	0.412	0.3680699	0.60258											
16	Min TEWI	0.486	0.457	0.4685313	0.38973											
17	Base design	0.472	0.453	0.445877	0.40129											
18																

Figure 14: Sheet 3 of the extended Excel-aided model for the optimised system

Comparison of Figure (14) with Figure (11) shows that Solver increased the value of C_i from 0.591 to 0.765 by adjusting the values of the four design variables as shown on Table (6). Figure (15) compares the modified values of the four optimised performance indicators obtained by Solver with those of the base design and those obtained by the MOO solution of MIDACO. The figure shows that the increments in the COP and exergetic efficiency obtained by Solver-TOPSIS solution are significantly higher than those of the solution obtained by MIDACO while the TEWI is significantly lower. These improvements are achieved by increasing the total cost rate which increased by 9.65%. (Actually, both the cost of electricity and the penalty for CO₂ emissions decreased, but the purchased equipment cost increased by 12%). With respect to the hot water, Table (6) shows that the method decreased the flow rate from 0.005 kg/s to 0.004 kg/s, but increased the exit temperature from 34.6°C to 35.99°C.

Table 6: Particulars of the base design, the MIDACO solution, and the Solver-TOPSIS solution

	Base design	MIDACO	Solver -TOPSIS
T_E [°C]	-20	-15	-15
T_C [°C]	40	41.439	38.010
P_{ic} [kPa]	331.256	351.239	377.971
\dot{m}_{water} [kg/s]	0.005	0.001	0.004
T_{w2} [°C]	34.61	33.29	35.99

Figure (16) compares the rates of exergy destruction in the different system components of the base design to those adjusted by the MIDACO solver and the Solver-TOPSIS technique. As the figure shows, the highest rate of exergy destruction for all three systems occurs in compressor 2 followed by compressor 1, throttle valve1, throttle valve 2 and then the condenser. By comparison, the rates of exergy destruction in the remaining three components are negligible. The figure also shows that the total rate of exergy destruction in the system optimised by the Solver-TOPSIS technique is less than the corresponding values of the base design and that optimised by the MIDACO solver.

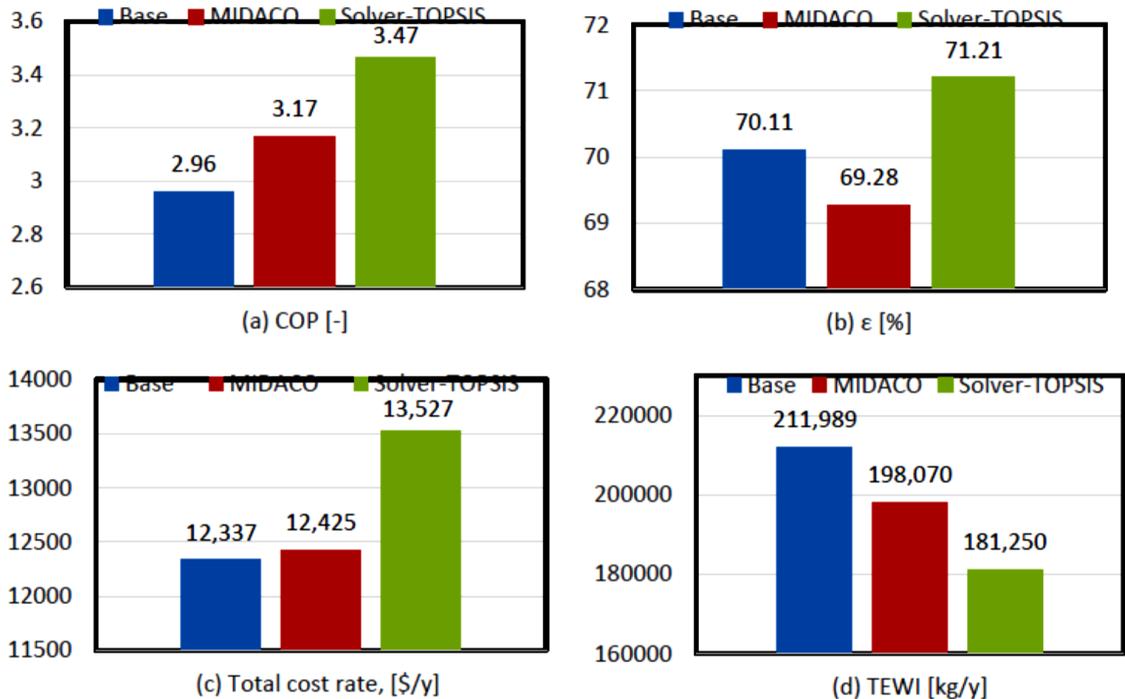


Figure 15: Comparison of the four key performance indicators for the base design with those obtained by MIDACO and the Solver-TOPSIS technique

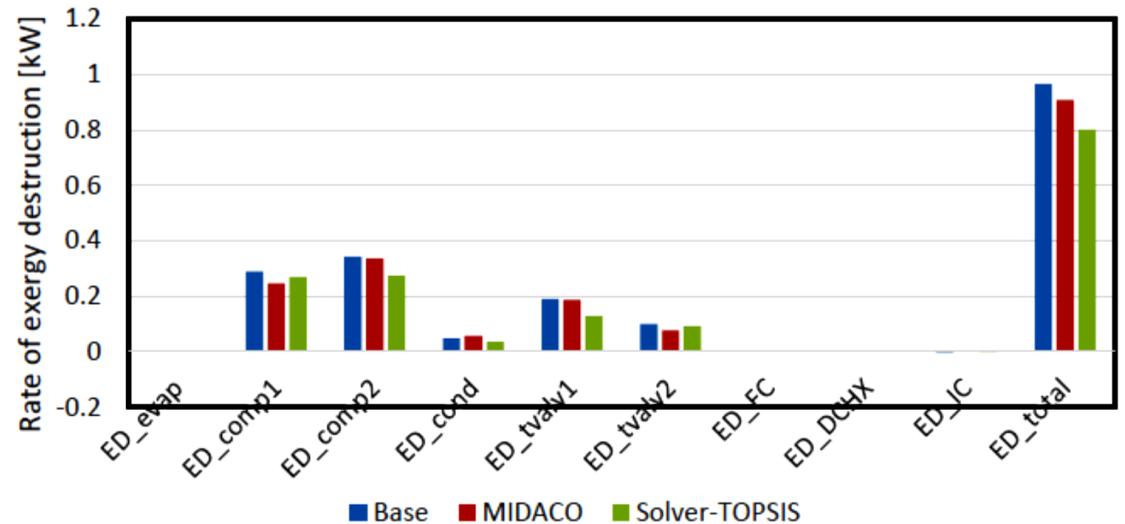


Figure 16: Exergy destruction rates in the base design and at the optimal solutions obtained by MIDACO and the Solver-TOPSIS technique

CONCLUDING REMARKS

This paper describes a method for utilising the TOPSIS decision-making technique with the single-objective Solver of Microsoft Excel for multi-objective optimisation analyses of multi-stage compression VCR systems. The method is applied to analyse an innovative two-stage system that incorporates an intercooler after the first compression stage and recovers the waste energy for producing hot water. Four single-objective solutions are first obtained by using Solver to maximise the COP and exergetic efficiency and minimise the total cost and TEWI. Four design variables are used in these analyses which are the evaporator and condenser temperatures, the isentropic efficiency of the compressor, and the water flow rate. Solver is then used to simultaneously satisfy the four objectives by using the TOPSIS method. Comparison of the optimised system obtained by this method with that obtained by using the MIDACO multi-objective solver shows that the proposed method leads to higher values of the system's COP and exergetic efficiency and a lower value of its TEWI at the expense of increasing the total cost rate.

List of Symbols and Abbreviations

c	-	Unit cost
\dot{C}	-	Cost rate
CC	-	Cooling capacity of the system
COP	-	Coefficient of performance
CRF	-	Capital-recovery factor
\dot{E}^D	-	Rate of exergy destruction
GWP	-	Global-warming potential
h	-	Enthalpy
i	-	Interest rate
MOO	-	Multi-objective optimisation
m	-	Mass of refrigerant
\dot{m}	-	Mass flow rate
N	-	Annual operation hours
n	-	Plant life time
\dot{Q}	-	Rate of heat transfer
s	-	Entropy
SOO	-	Single-objective optimisation
T	-	Temperature
TEWI	-	Total equivalent warming index
\dot{W}	-	Work
x	-	Quality or dryness fraction of refrigerant

Greek letters:

α	-	Refrigerant recycling factor
β	-	Electricity regional conversion factor
ε	-	Exergetic efficiency
η	-	Isentropic efficiency of compressor
μ	-	Emission factor
ϕ	-	Maintenance factor,

Subscripts:

CO_2	-	Carbon-dioxide
$elec$	-	Electricity
r	-	Refrigerant
ref	-	Refrigerant
w	-	Cooling water

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