

ARCHITECTURAL GEOMETRY REIMAGINED: THE ROLE OF BEZIER AND PARAMETRIC CURVES IN MODERN DESIGN WORKFLOW

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المخلص

يستكشف هذا البحث الدور المتطور للانحناء الرياضي ونظرية منحنى بيزيه في التصميم المعماري المعاصر، مع التركيز على نمذجتهما الحاسوبية وآثارهما الإنشائية. ويحلل البحث منحنيات الدرجة الثانية والمنحنيات البارامترية، بما في ذلك الانحناء الغوسي، ويسلط الضوء على تكاملها في سير عمل التصميم الرقمي من خلال منصات مثل Rhino/Grasshopper و Autodesk Revit. ويتناول البحث التطورات الحديثة في مشتقات بيزيه المنطقية، وتقاطعات المنحنيات الذاتية، والهندسة الجاهزة للتصنيع من خلال مساهمات رياضية جديدة. وتُظهر دراسات حالة، مثل مشروع "مركز الملك عبد الله للدراسات والبحوث البترولية" KAPSARC، التطبيق العملي للمورفولوجيا والتحسين الطوبولوجي القائم على الأداء والمحرك بالذكاء الاصطناعي. ويؤكد البحث على كيفية مساهمة الأدوات الرقمية والدقة الرياضية معًا في دفع عجلة الابتكار في التعبير الجمالي، والأداء الإنشائي، والاستدامة في الممارسة المعمارية.

ABSTRACT

This paper explores the evolving role of mathematical curvature and Bézier curve theory in contemporary architectural design, with a focus on their computational modeling and structural implications. It analyzes second-order and parametric curves, including Gaussian curvature, and highlights their integration in digital design workflows through platforms like Rhino/Grasshopper and Autodesk Revit. Recent advancements in rational Bézier derivatives, curve self-intersections, and fabrication-ready geometries are examined through new mathematical contributions. Case studies such as the "King Abdullah Petroleum Studies and Research Center" (KAPSARC) project demonstrate the real-world application of AI-driven, performance-based morphology and topological optimization. The research underscores how digital tools and mathematical rigor together drive innovation in aesthetic expression, structural performance, and sustainability in architectural practice.

KEYWORDS: Architectural Geometry, Topological Optimization, Rhino/Grasshopper, Curve Fabrication-Ready Geometry.

INTRODUCTION

The relationship between architecture and mathematics has long underpinned the creation of expressive, structurally sound built environments. For centuries, classical geometric forms—such as circles, ellipses, and parabolas—have shaped the design of domes, vaults, and acoustically optimized spaces. In today's digital era, these enduring

principles have evolved into advanced parametric systems and free-form surfaces governed by precise mathematical frameworks. Among these tools, Bézier curves and spline-based models grant architects refined control over curvature continuity, form rationalization, and fabrication precision. The emergence of AI-driven design tools and topological optimization techniques now enables architectural forms to adapt dynamically to structural loads and environmental conditions. This study draws on foundational research into second-order curves, surface curvature, and Bézier theory, while also integrating recent investigations into practical challenges such as self-intersections and derivative precision. Through diagrammatic exploration, classification of spatial curvature, and computational overlays, this work positions curvature not just as an aesthetic device but as a fundamental force behind performance-driven architecture. The mathematical shaping of architectural form has deeply influenced the evolution of the built environment. Historically, architects employed geometric constructs like circles, ellipses, and parabolas for their visual elegance and their inherent structural and functional advantages. With the rise of computational design, these traditional forms have given way to more complex geometries defined by algorithms and parametric logic. Bézier curves and spline models have become essential in crafting free-form, digitally fabricated architecture. Spanning historical precedents to contemporary AI-assisted workflows, this paper offers a comprehensive perspective on curvature as both a design language and a driver of performance in modern architectural practice. Earlier studies have laid the groundwork for understanding curved geometries in architecture, exploring topics such as second-order curves in dome construction, parametric design methodologies, and structural shell behavior rooted in Gaussian curvature. This section consolidates these foundational insights, demonstrating how they continue to inform and shape current computational approaches in architectural design.

METHODOLOGY

This research employs a qualitative and diagrammatic approach by analysing historical references, mathematical derivations, and architectural case studies. Equations were selected based on their foundational role in describing curvature and digital modelling logic. Graphical overlays were created to compare different curve types. Diagrams were generated using Rhino/Grasshopper for spatial curvature modelling.

The interplay between architecture and mathematics has continuously evolved, demonstrating an enduring partnership that shapes human environments. Historically, architects relied heavily on intuitive spatial and structural exploration, evident from classical domes and Gaudí's innovative catenary arches, to contemporary complex geometries. However, with the advent of computational tools, architectural geometry has transformed dramatically, becoming an explicitly defined and systematically explored field. Today, sophisticated digital modelling techniques, notably Bézier and parametric curves, enable architects to achieve unprecedented levels of aesthetic complexity, structural optimization, and functional efficiency.

CLASSICAL CURVES IN ARCHITECTURAL DESIGN

Second-Order Curves [1]

Second-order curves include circles, ellipses, parabolas, and hyperbolas. These shapes are derived from quadratic equations and have been used extensively in

traditional architecture. A circle, which makes it suitable for domes and arches where uniform force distribution is desired. The ellipse was used in Roman and Baroque architecture for its ability to direct acoustic and visual focus in amphitheatres and cathedrals. Parabolas concentrate light or sound energy, making them ideal for vaulted ceilings in sacred architecture. Hyperbolas offer aesthetic dynamism and were favoured in futuristic or high-tech architectural expressions.

Catenary Curves

Catenary were employed by Antoni Gaudí in his gravity-based designs to align with natural compression paths in structures. They distribute compressive forces more efficiently than parabolas and are evident in the design of suspension bridges, cathedral arches, and tensile fabric roofs. In modern architecture, catenary principles inform the design of long-span shell structures and kinetic facades that react to environmental loads. For comparison of catenary curve and parabola curve, see Figure (1).

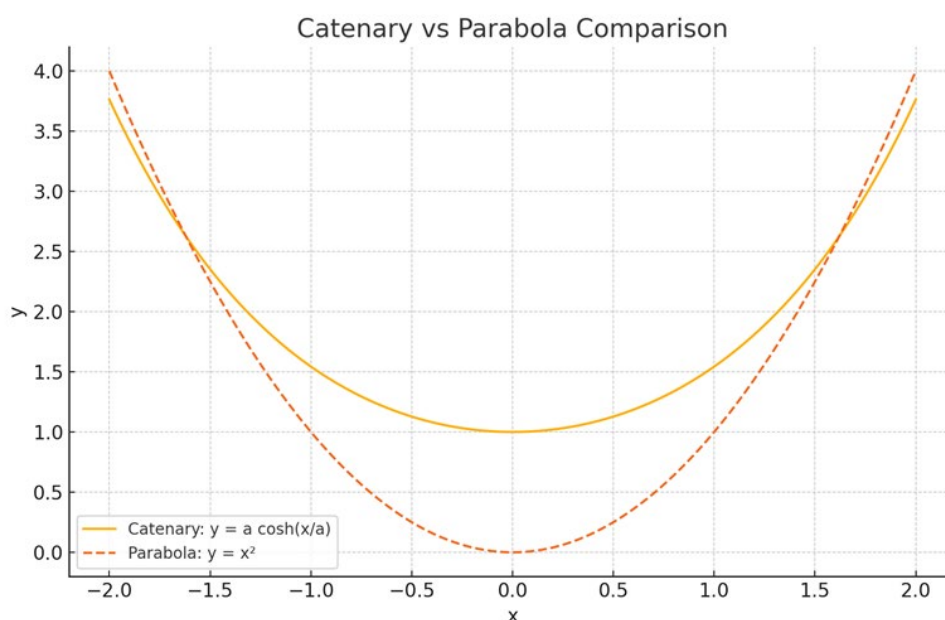


Figure 1: Comparison of the shapes of a catenary and a parabola, showing differences in load-bearing behaviour.

Surface Curvature and Gaussian Principles

Gaussian curvature is defined as [2]

$$K = (f_{xx} f_{yy} - f_{xy}^2) / (1 + f_x^2 + f_y^2)^2 \quad (1)$$

This property is vital in 3D form-making, such as domes (positive K), hyperboloids (negative K), or planes (zero K). Understanding Gaussian curvature helps designers determine how materials will bend or resist deformation. It enables rational panelling of curved surfaces, structural efficiency in shells, and innovative uses of double curvature in membrane structures. Additionally, advancements in computational

design allow architects to simulate and manipulate Gaussian curvature to create visually striking yet structurally sound forms [2].

PARAMETRIC AND FREE-FORM CURVES ANALYSIS IN ARCHITECTURE

Parametric Equations and Design Logic

Parametric design allows geometry to be defined by variables and equations, enabling a flexible and responsive design workflow. Unlike static modelling, parametric modelling establishes relationships between form and data, empowering architects to quickly iterate, optimize, and adapt shapes to site constraints or performance criteria. For example, a sinusoidal curve can be adjusted in real-time by altering the amplitude or frequency based on wind simulations or sun path analysis.

Tools such as Rhino3D with Grasshopper, Autodesk Revit with Dynamo, Maya MEL, and Houdini are now standard in architecture and urban design studios. These platforms offer visual scripting environments, allowing designers to program geometric behaviour without writing raw code [3]. Common mathematical expressions used in parametric architecture include:

- Sine waves for building skins or shading systems
- Logarithmic spirals for ramps or spiral staircases
- Helical surfaces in roof trusses or bridges
- Parametric meshes, used in complex pavilions

These equations support algorithmic design principles, contributing to the aesthetics and performance of free-form architecture.

BÉZIER CURVES AND SPLINES IN ARCHITECTURAL MODELLING

Recent advancements have addressed the challenges posed by self-intersections in cubic Bézier curves. Yu et al. [4] introduced an efficient algorithm that transforms the self-intersection problem into a solvable linear system, enhancing the reliability of Bézier curve applications in architectural modelling. Additionally, Raseli et al. [5] provided a comprehensive study on constructing cubic Bézier curves using expanded basic functions, emphasizing their properties and applications in computer-aided geometric design. This foundational understanding aids architects in accurately modelling complex geometries.

Bézier curves, first introduced by Pierre Bézier in automobile design, are now fundamental to architectural geometry. A cubic Bézier curve is mathematically defined as [2]

$$B(t) = (1 - t)^3 P_0 + 3(1 - t)^2 t P_1 + 3(1 - t) t^2 P_2 + t^3 P_3 \quad (2)$$

where $0 \leq t \leq 1$.

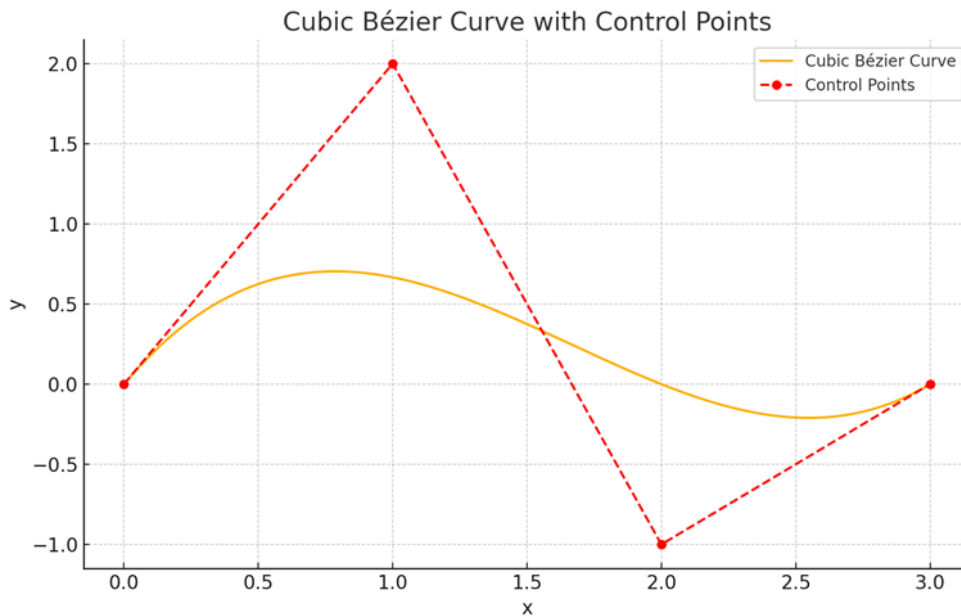


Figure 2: Annotated Bézier curve analysis using four control points, illustrating control structure, curve shape, and tangent vectors.

Here, P_0 through P_3 are control points that influence the curve's shape. Architects use Bézier curves for modelling:

- Free-form façades (e.g., complex curtain walls)
- Tessellated surfaces (e.g., ETFE or aluminium panels)
- Curved interior features (e.g., balustrades, furniture)
- Fabrication-ready profiles for CNC milling or robotic construction.

These curves are often extended into Bézier splines—a sequence of Bézier segments joined with continuity constraints (C^1 or C^2), ensuring smooth transitions across curves.

In more advanced applications, Bézier models can be transformed into NURBS (Non-Uniform Rational B-Splines) to handle high-precision tasks such as automotive surface design or fluid architectural envelopes.

ARCHITECTURAL TOOLS AND PARAMETRIC CASE APPLICATIONS

Many iconic structures exemplify the use of parametric and Bézier-based curves:

- Yas Hotel (Abu Dhabi): Designed using parametric skins based on curvature variation across the façade
- BMW Welt (Munich): A double-curved form developed using NURBS to accommodate structural and spatial fluidity
- Al Bahar Towers (Abu Dhabi): Adaptive shading system designed with a parametric algorithm reacting to solar radiation

These examples show how parametric systems link geometry, performance, and material behaviour, forming a new paradigm in design thinking.

Platforms like Karamba3D and Ladybug Tools extend this logic into structural and environmental analysis, allowing geometry to react to dynamic inputs such as wind loads, daylighting, or thermal variation.

MATHEMATICAL AND PRACTICAL BENEFITS

Parametric and Bézier-based modelling enhances architectural performance in several ways:

- Adaptability: Designs can quickly be modified by changing underlying variables
- Structural Efficiency: Forms can be analysed and refined to minimize material waste
- Fabrication Readiness: Smooth curves reduce mechanical stress during fabrication and simplify toolpath creation
- Data Responsiveness: Architectural forms can be linked to real-time or historical datasets (e.g., GIS, sensor networks)

Moreover, the continuity properties of Bézier curves (smooth derivatives and predictable curvature) are crucial in both visual aesthetics and construction tolerances.

EVOLUTION OF CURVATURE APPLICATIONS IN ARCHITECTURAL GEOMETRY

This timeline graphically represents the historical evolution of curvature usage in architectural geometry, from classical circles and ellipses to contemporary fabrication-aware strategies integrating digital tools and optimization techniques.

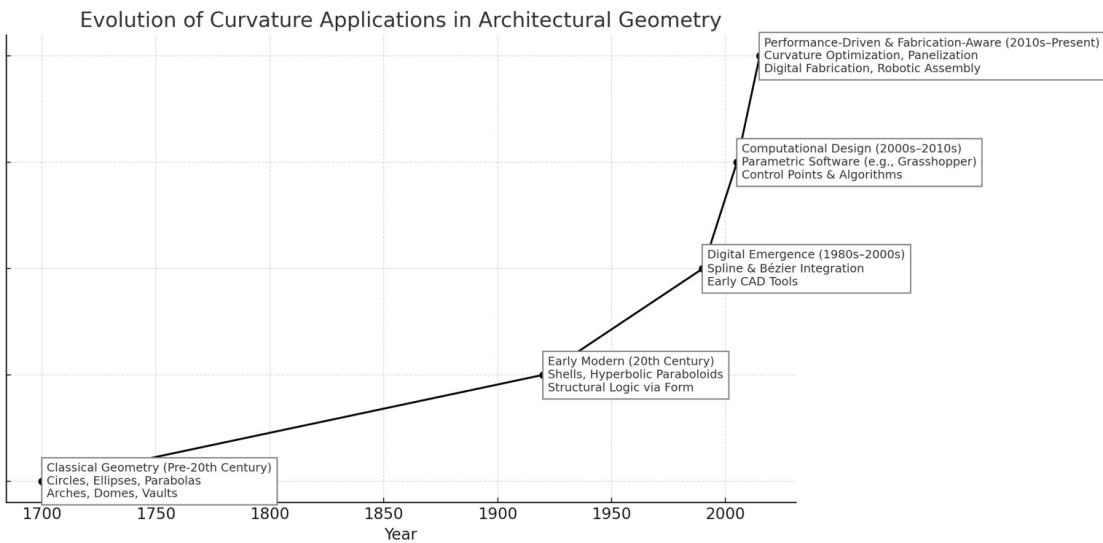


Figure 3: Timeline representing the historical evolution of curvature usage in architectural geometry.

SPATIAL CURVATURE IN ARCHITECTURAL GEOMETRY

These figures illustrate the three fundamental types of curvature in architectural spatial design. Understanding these helps in selecting appropriate geometries for structural performance, material efficiency, and visual aesthetics:

Positive Curvature (Dome)

Dome-shaped forms exhibit positive Gaussian curvature, often used in shells and domes for even stress distribution.

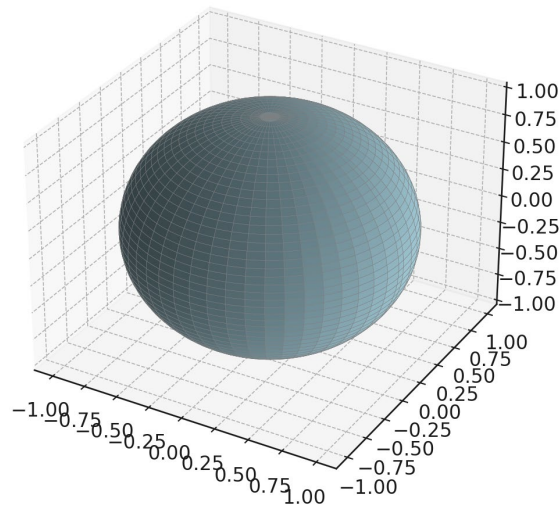


Figure 4: Positive Gaussian curvature.

Negative Curvature (Saddle Surface)

Hyperbolic paraboloids and saddle surfaces represent negative curvature, applied in tensile roofs and expressive shell structures.

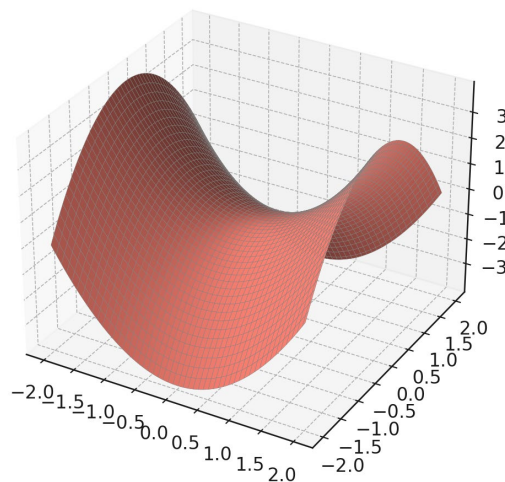


Figure 5: Negative curvature.

Zero Curvature (Flat Plane)

Flat planes exhibit zero curvature and are common in conventional slab and wall construction.

Zero Curvature (Flat Plane)

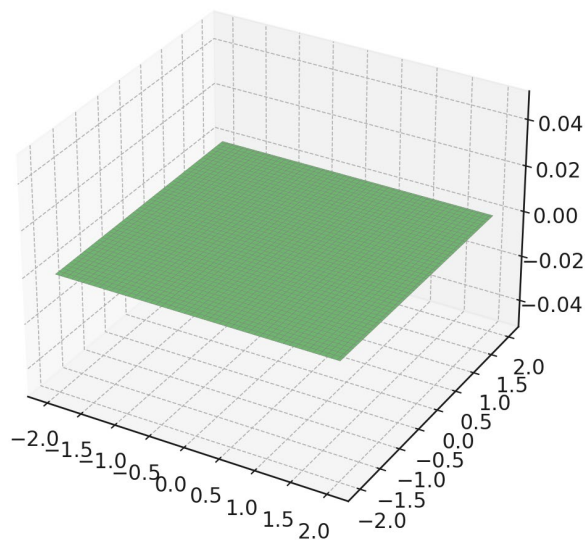


Figure 6: Zero Curvature.

Comparative Relationship of Curvature Types

This bar graph illustrates the relative Gaussian curvature values for three geometric surface types used in architectural design: negative curvature for hyperbolic (saddle) forms, zero curvature for flat planes, and positive curvature for dome-like structures. This comparison aids in understanding spatial form behaviour in terms of structural efficiency and visual impact [6].

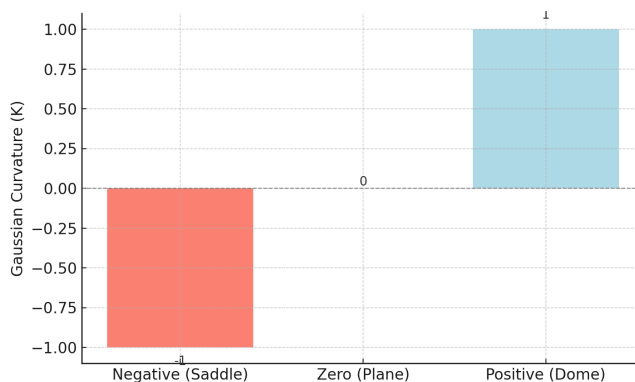


Figure 7: Relative Gaussian curvature values for three geometric surface types.

OVERLAY ANALYSIS OF ARCHITECTURAL CURVE TYPES

Advanced Mathematical Considerations in Bézier Curve Applications

Shi [7] addressed limitations in existing derivation formulas for rational Bézier curves, presenting a new formula that accurately computes higher-order derivatives

beyond the curve's degree. This advancement is crucial for precise curvature analysis in architectural designs involving rational Bézier curves.

Furthermore, Huylebrouck [8] explored the application of curve fitting techniques in architecture, demonstrating how mathematical methods can be employed to analyse and replicate architectural forms. This perspective underscores the intersection of mathematics and architectural aesthetics.

This overlay graph demonstrates the comparative geometry of various curve types used in architectural design evolution. It includes:

- Classical curves (e.g., circular arcs)
- Parametric curves (sinusoidal)
- Digitally emergent curves (modulated exponential forms)
- Computationally derived curves (nonlinear cubic transformations)
- Performance-driven curves (optimized parabolic arches)

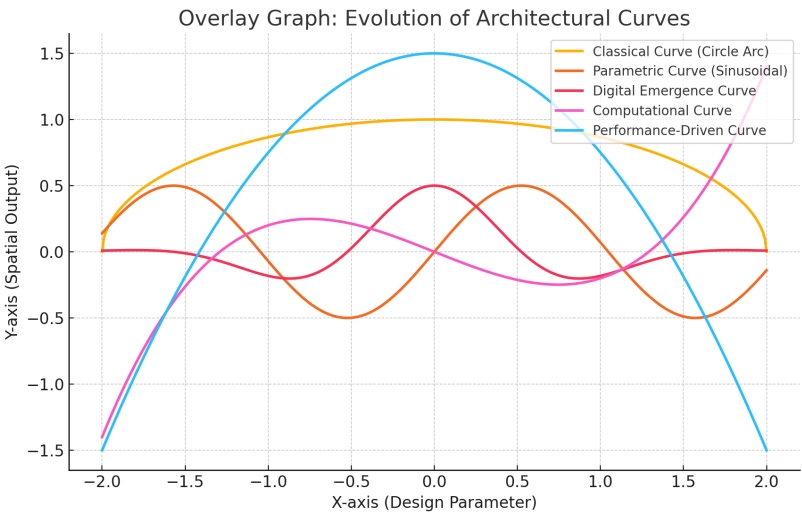


Figure 8: Overlay Analysis of Architectural Curve Types.

FUTURE DIRECTIONS IN CURVATURE-BASED DESIGN

A compelling case study is Zaha Hadid Architects' use of AI-driven design tools in the design of the KAPSARC building. The team used generative algorithms to evolve curved geometries in response to environmental simulations. Topological optimization tools like Autodesk Fusion 360 and AI-assisted tools helped reduce material usage while maintaining structural performance. These methods are increasingly being adopted in real-time design workflows where architectural form evolves with input from performance constraints.

Emerging applications include AI-generated curvature forms, performance-driven topological optimization, and real-time sensor-based adaptations of curvature for climate-responsive structures. These directions align mathematical abstraction with real-world sustainability goals.

CONCLUSION

Curvature-based design has transitioned from aesthetic experimentation to a performance-cantered discipline shaped by mathematical precision and digital responsiveness. This study has traced the lineage of curved geometries from classical constructs to computationally defined surfaces, emphasizing the roles of Bézier curves, Gaussian curvature, and parametric equations. Integrating rational Bézier derivatives and addressing curve continuity and self-intersection improves modelling reliability in complex spatial forms.

The use of AI-assisted design tools and topological optimization exemplified by projects like Zaha Hadid Architects' KAPSARC demonstrates how curvature is now adaptive, data-informed, and fabrication-aware. Parametric workflows empower architects to create sustainable, structurally efficient, and visually dynamic environments while minimizing waste and maximizing precision. As architectural geometry becomes increasingly informed by real-time environmental and structural data, the mastery of curvature mathematics will be central to the discipline's future.

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