

SPECTRAL CHARACTERIZATION OF FLAME INSTABILITY INDUCED BY ACOUSTICALLY EXCITED FUEL IN DIFFUSION FLAMES WITH DIFFERENT BURNER DIAMETERS

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التوصيف الطيفي لعدم استقرار اللهب الناتج عن الوقود المثار صوتياً في لهب الانتشاري بأقطار موقد مختلفة

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الملخص

في هذه الورقة تم دراسة تأثير الوقود المثار صوتياً على استقرار اللهب الانتشاري (diffusion flames) في أنظمة الاحتراق المستمر، مع التركيز على قطر الحارق. تصميم الحارق يؤثر بشكل كبير على استقرار الاحتراق، لا سيما فيما يتعلق بعدم الاستقرار الحراري السمعي. حيث استخدم نظام إثارة صوتية، يتضمن مولد إشارة ومضخم، لإثارة الوقود عند ترددات وشدة صوتية محددة. تم الحفاظ على معدل تدفق الوقود ثابتاً مع تغيير أقطار الحارق (0.5 مم، 0.75 مم، و1.0 مم) لتقييم تأثيرها على ديناميكيات اللهب. تم مراقبة سلوك اللهب باستخدام خلية ضوء معايرة لالتقاط انبعاثات اللهب الضوئية الكيميائية (chemiluminescence)، وتم تحليل الإشارات وحساب الارتباط الذاتي (auto-correlation) والتحليل الطيفي (power spectrum) لها. أظهرت النتائج أن عدم استقرار اللهب كان ثابتاً في كل أقطار الحارق، مع ملاحظة أقوى عدم الاستقرار للهب كان عند تردد الإثارة السمعية 90 هرتز، وأقوى استقرار عند 150 هرتز، وكان اللهب غير مستقر جزئياً عند 175 هرتز. أظهرت النتائج أن الحارقات ذات الأقطار الأصغر كانت أكثر عرضة لعدم الاستقرار، وذلك بسبب ارتفاع سرعة تدفق الوقود عبر فوهة الحارق. وقد بدا هذا عدم الاستقرار واضحاً من خلال ضعف ترابط الإشارة وازدياد تعقيد التحليل الطيفي مقارنة بالحارقات الأكبر قطرًا. من خلال التحليل الطيفي للهب المستقر يوجد به تردداً مهماً واحداً، بينما في اللهب الغير مستقر قمم تردد متعددة. تسلط هذه النتائج الضوء على الدور الحاسم لإثارة الصوتية وهندسة الحارق في التحكم في استقرار اللهب، مما يوفر رؤى مهمة لتصميم وتحكم أنظمة الاحتراق.

الكلمات المفتاحية: عدم الاستقرار الحراري السمعي، انبعاثات اللهب الضوئية الكيميائية، التحليل الطيفي، اللهب الانتشاري.

ABSTRACT

This study investigated the impact of acoustically excited fuel on the instability of diffusion flames in continuous combustion systems, focusing on burner geometry. Its design significantly influenced combustion instability, particularly thermo-acoustic

instabilities. A controlled acoustic excitation system was employed to introduce fuel perturbations at specific frequencies and acoustic intensities. The experiment at constant fuel flow rate was conducted while varying burner diameters (0.5 mm, 0.75 mm, and 1.0 mm) to assess their influence on flame dynamics. Results indicated that flame instability was consistent across all configurations, with the highest instability observed at 90 Hz, flame strongly stable at 150 Hz, and intermediate behavior at 175 Hz. Smaller burner diameters exhibited greater instability, as evidenced by reduced signal correlation and increased spectral complexity. Stable flames exhibited a single dominant frequency, whereas unstable flames showed multiple distinct frequency peaks. These findings highlight the critical role of acoustic excitation and burner geometry in governing flame stability, offering insights relevant to the design and control of combustion systems.

KEYWORDS: Thermoacoustic instability, Chemiluminescence, Spectral analysis, Diffusion flame.

INTRODUCTION

The phenomenon of flame instability in the combustion chambers of heat engines is one of the most critical challenge that must be addressed, due to its strong impact on engine performance affecting thermal efficiency, environmental pollution, and fuel consumption. This is especially important in engines operating with continuous fuel injection systems. One of the most significant types of instability is thermo-acoustic instability, which results from the interaction between pressure oscillations inside the combustion chamber and the heat release oscillations. Therefore, the burner's shape and diameter contribute to the amplification or reduction of this phenomenon. Under certain conditions, the burner plays a significant role in either intensifying or suppressing thermo-acoustic flame instability. Many studies have focused on this phenomenon to understand its occurrence and to develop methods to reduce or avoid it by using either passive control systems or active control systems. The size of the burner is particularly important in mitigating the phenomenon when using passive combustion control methods [1-7].

Among the most important studies are those that examined burner behavior through changes in the burner position or by extending its nozzle, with the aim of reducing the coupling between pressure oscillations and the heat oscillations emitted from the combustion process, and its relation to thermo-acoustic instability. Many researchers have focused on this topic, particularly regarding methods to suppress this phenomenon using passive control systems. Among these studies is the work by Hermann et al. [8], who modified hydride burners in a Siemens Vx4.3A gas turbine to reduce combustion instabilities caused by interactions between acoustic waves and flow. Welding cylindrical burner outlets (CBOs) to extend convective time lag to destabilizing self-excited oscillations effectively delayed the combustion response. When paired and inclined at 10°, these extensions formed asymmetric burner outlets (ABOs), which disrupted coherent flow structures and shifted the combustion zone downstream, further increasing time lag. As more ABOs were added, damping improved, and turbine power rose by 7%. Similar gains were achieved with asymmetric CBOs, reaching a 9% power increase with 20 installed units.

Stefanie et al. [9], presented a procedure aimed at selecting the most stable burner geometry for a given combustor. Developed a generic burner design featuring significant variability in dynamic flame response depending on two geometrical

parameters. Through experimental measurements and physics-based parametric models, they demonstrated that these model parameters correlate uniquely with variations in burner geometry, allowing for interpolation and optimization of burner design to enhance thermo-acoustic instability, and demonstrating that burner design parameters, including burner diameter, significantly influence flame dynamics and the combustor's thermo-acoustic instability. Zhou et al. [10]. experimentally investigated how modifications in burner geometry specifically, the length of the inlet section and the presence of separation plates affect the response of a non-premixed swirl flame under acoustic excitation and examine how different burner geometries, including variations in burner diameter, affect the response of non-premixed flames to acoustic excitation. The findings highlight the importance of burner design in influencing flame dynamics and mitigating combustion instabilities. Emadi, et al. [11]. investigate the thermo-acoustic instability characteristics of low-swirl burners with different diameters. The research finds that burner diameter influences the frequency and nature of acoustic modes, which are crucial in understanding and controlling thermo-acoustic instabilities. Dustin et al. [12]. present a novel actuator concept that utilizes sub-breakdown electric fields and the electro-hydrodynamic effect to stabilize a laminar flame. This actuator enables variable distortion of the flame and bidirectional forcing of its heat release. The actuator was combined with a simple feedback controller to suppress a thermo-acoustic instability, reducing the instability's sound pressure level by 27 dB in less than 60 milliseconds. The system required only 40 mW of power to stabilize a 3.4kW thermal power flame, highlighting its potential for practical applications in combustion systems. Thannickal, et al. [13] investigate a trim adjustment active control system based on twin variable volume Helmholtz resonators for controlling combustion noise in a methane-air combustion chamber. The results demonstrate that the twin resonator system can effectively control combustion noise in open-loop conditions across a range of equivalence ratios, outperforming single resonator systems. The paper also outlines a practical scheme for implementing this system in open-loop applications and suggests further investigation for closed-loop suppression of thermo-acoustic instability. Premchand, et al. [14] propose a framework utilizing Lagrangian Coherent Structures (LCS) to identify optimal locations for passive control of tonal sound generated during thermo-acoustic instability. Experiments conducted in a laboratory-scale bluff-body stabilized turbulent combustor revealed that injecting a steady micro-jet of air at the identified optimal location resulted in a significant reduction in sound amplitude. This approach offers a promising strategy for mitigating thermo-acoustic instabilities in combustion systems. Kojourimanesh, et al. [15]. apply system theory methods to introduce a new analysis methodology based on the stability criteria of active two-ports to the problem of thermo-acoustic instability in combustion appliances. They utilize the analogy between thermo-acoustics of combustion and the small-signal operation of microwave amplifiers. The paper introduces notions of unconditional and conditional stabilities of an active two-port, representing a burner with flame, and analyzes these concepts in the context of thermo-acoustic systems. The study provides algebraic techniques and properties of Möbius transformations to derive necessary and sufficient conditions for the stability of linear active two-port systems in combustion applications. Qi, Y. et al. [16]. investigate the combustion instability characteristics of a swirling flame with an outer flame under different operating conditions, focusing on how burner design affects instability behavior.

Due to the significant impact of the burner on the phenomenon of combustion instability, this study will examine flame instability through spectral analysis and the correlation of the chemiluminescence signal from a diffusion flame with different burner diameters. The aim is to contribute to a deeper understanding of the phenomenon and to support the selection of an appropriate control system for managing flame instability. This study aims to investigate the relationship between burner diameter and flame stability under the influence of acoustically excited fuel, providing insights into how burner geometry contributes to the suppression or amplification of thermo-acoustic instability.

This work provides a unique contribution by examining how acoustically excited fuel interacts with burner geometry to influence thermo-acoustic instability in a free jet diffusion flame. It investigates the effect of very small burner diameters (0.5, 0.75, and 1.0 mm) under controlled acoustic excitation and applies spectral and auto-correlation analysis of chemiluminescence signals to quantify instability across configurations. The study identifies a clear frequency-dependent behaviour, with a strong instability peak at 90 Hz and a stable response at 150 Hz, revealing how acoustic forcing shapes flame dynamics. Overall, this study contributes new experimental evidence on the combined role of burner geometry and acoustic fuel excitation in thermo-acoustic instability, supporting improved design and control of continuous combustion systems.

The purpose of this study is to determine how burner diameter and acoustic excitation frequency affect the stability and spectral behavior of a free jet propane diffusion flame.

EXPERIMENTAL SETUP

This experiment investigates the dynamics of a free jet diffusion flame under atmospheric pressure, excited by acoustic perturbations. Figure (1) presents the schematic of the experimental setup. The core components include a combustion unit, an acoustic excitation system, and a chemiluminescence measurement system integrated with high-speed data acquisition. The combustion system consists of a single copper burner tube (0.5 cm inner diameter) with interchangeable orifices (0.5, 0.75, and 1.0 mm). It is connected to a compressed propane cylinder via a fuel supply line equipped with a rota meter and control valves for precise flow regulation. Propane is used as the fuel, and its flow rate is adjustable to control the flame characteristics. The acoustic excitation unit comprises a PeakTech signal generator connected to a small loudspeaker operating within the 20–1000 Hz range. The loudspeaker is enclosed in a 15 cm diameter × 10 cm high cylindrical box to act as a monopole source, enabling controlled acoustic excitation of the fuel flow. Flame dynamics are monitored through chemiluminescence emission using a light cell connected to a data acquisition system with a sampling rate of up to 1.25 MS/s. The acquired signals are processed and analyzed using LabVIEW software to capture flame response under acoustic forcing.

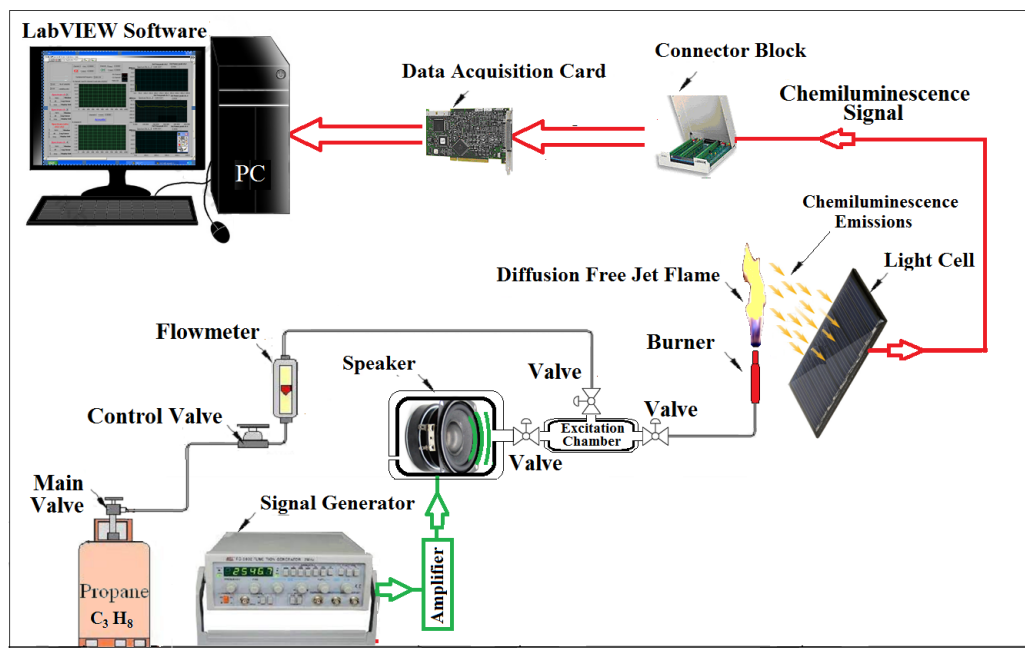


Figure (1): Overview of the Experimental Setup.

RESULTS AND DISCUSSION

For this investigation, three burner diameters were chosen: 0.5, 0.75, and 1.0 mm. The fuel flow rate was maintained constant at 100 ml / min laminar condition to ensure that the burner diameters was the primary variable influencing the flame dynamics. By varying the burner diameter, it was observed how this parameter affects the flame's stability under controlled experimental conditions.

The flame's oscillations or fluctuations were measured in the form of fluctuating chemiluminescence. chemiluminescence refers to the light emitted by the flame during the combustion process. A calibrated light cell was used to measure the oscillation of the light intensity emitted from the flame by tracking the time-dependent variations of the chemiluminescence signal. After recording the signal, a spectral analysis was performed to determine the frequencies associated with the flame oscillations, with the aim of assessing its stability or susceptibility to instability. This helps in identifying the dominant frequencies at which the flame oscillates, which is important for understanding the instability patterns in the combustion process. The signal's auto-correlation was also computed, which provides insight into the temporal relationship of the signal with itself. A high correlation suggests a regular, stable pattern, while a low correlation indicates more random, unstable behavior. The degree of correlation is thus an important indicator of flame instability. To further explore the effects of different frequencies on flame instability, the fuel supplying the diffuse flame was excited at frequencies ranging from 50 Hz to 200 Hz using a signal generator. This excitation was applied through an acoustic amplifier, which controlled the amplitude of the sound. By adjusting the sound amplitude, the researchers were able to determine the specific sound pressure level at which the flame would become unstable and eventually extinguish.

Figure (2) illustrates the relationship between the sound amplitude and the flame excitation. This figure shows the voltage amplitude of the sound used to excite the fuel supplying the flame. The data collected for this figure were recorded before flame extinction occurred, allowing the researchers to observe the flame's response to

different sound amplitudes without crossing the threshold where the flame would extinguish. The data confirmed that larger burner diameters require higher sound amplitudes to reach the instability threshold, further indicating that they offer more stability than smaller diameters.

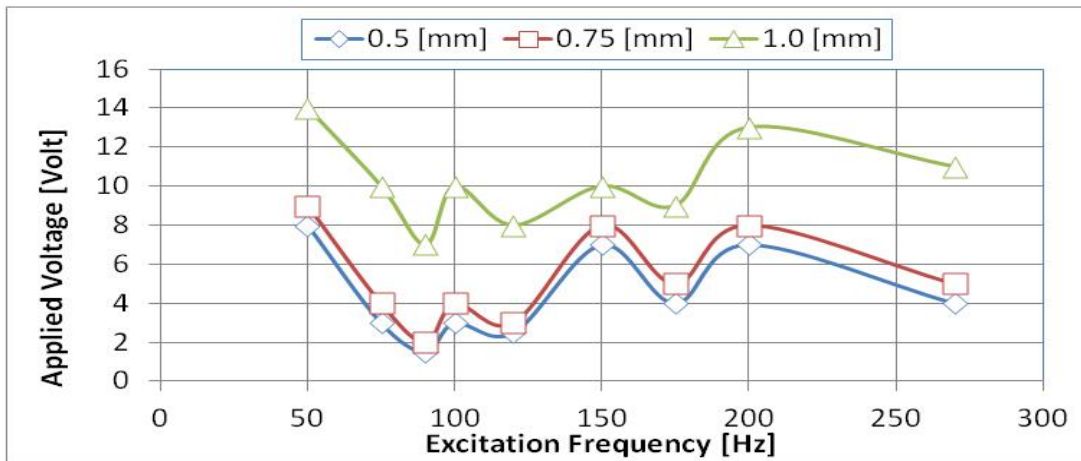


Figure (2): Fuel excitation frequency as a function of applied voltage at different burner diameters.

Figure (3) provides a visual representation of the effect of burner diameter on flame behavior. It shows how the burner diameter influences the required voltage amplitude of sound to excite the fuel at different frequencies. This figure helps to demonstrate how the larger the burner diameter, the more resistant the flame becomes to instability caused by external excitations, like sound waves.

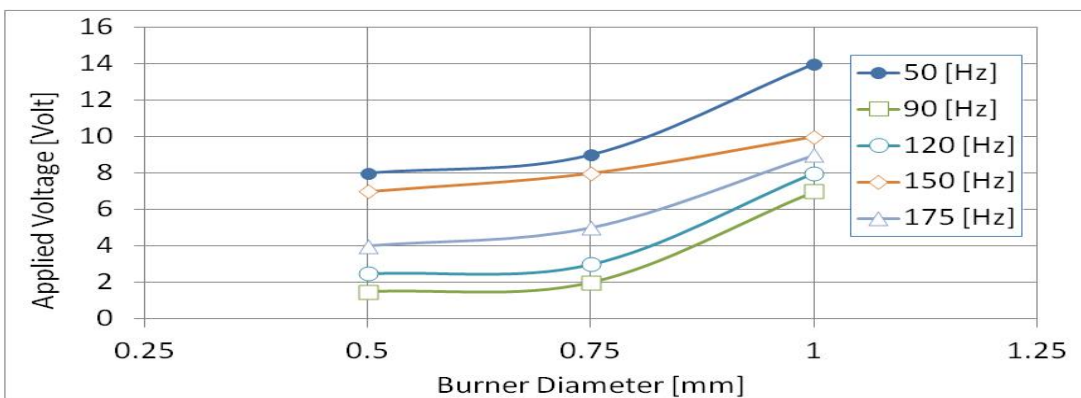


Figure (3): Burner diameter as a function of applied voltage at different excitation frequencies.

In this study, the relationship between the burner nozzle diameter and flame instability was explored by analyzing the chemiluminescence emission signal from the flame. chemiluminescence is the light emitted as a result of chemical reactions, particularly those occurring in flames, and it serves as a useful indicator of the flame's behavior and stability.

The experiment focused on analyzing the chemiluminescence emission from flames under different nozzle diameters, 0.5, 0.75, and 1.0 mm. These nozzle diameters were chosen to observe how changes in burner size affect flame behavior, especially in terms

of instability. The flame instability was studied under both stable and unstable conditions, which were controlled by varying parameters such as the fuel excitation frequency (the frequency at which fuel is introduced to the flame).

From earlier studies, the most pronounced flame instability was observed at a fuel excitation frequency of 90 Hz, consistently across all nozzle sizes. At this frequency, the highest level of instability was observed, indicating that the burner diameter was not a significant factor in determining the flame's instability mode. This is logical, as the primary factor influencing the stability characteristics is the length of the tube between the excitation unit and the other end where the burner is mounted. The standing wave formed inside the tube governs this behavior. In this experiment, the tube length corresponded to the first resonant frequency of the tube, which is 90 Hz, leading to the emergence of this instability mode.

This frequency was thus chosen as a reference point for studying flame instability in more detail. Additionally, the experiment also considered the frequency of 150 Hz, which produced the most stable flame behavior, and a third frequency of 175 Hz, which exhibited intermediate instability compared to 90 Hz.

To investigate the influence of acoustic on flame instability, the intensity of the sound emitted by the loudspeaker (which excites the fuel) was varied. The sound intensity was first set to zero (0 V), meaning no excitation, and gradually increased. As the sound intensity increased, the flame began to show variations in its behavior. The intensity was increased until the flame reached a point of near extinction, i.e., the flame was close to being extinguished due to the excessive excitation or instability caused by the sound waves.

The calculation of the auto-correlation of chemiluminescence signals is one of the key elements in the analysis process. This mathematical technique is used to measure how similar a signal is to itself over time, allowing for the assessment of the signal's regularity and stability. A strong auto-correlation indicates that the signal exhibits clear regularity, which is evidence of a stable flame with a periodic emission pattern, resembling the theoretical behavior of a sinusoidal signal. Conversely, when the auto-correlation coefficient is low and the plot rapidly decays toward zero, this indicates that the signal is random and irregular, which is a sign of flame instability and fluctuating combustion behavior.

In addition to auto-correlation, spectral analysis was performed on the chemiluminescence emission signals. Spectral analysis breaks down the emission signal into its constituent frequencies, providing a detailed view of the different oscillations present in the flame's emission. This is useful for identifying any periodic or oscillatory behavior that could indicate instability. By examining the spectral composition of the emission signal, the study could determine whether the flame exhibited distinct frequency components that correlate with stable or unstable behavior.

The results of the experiment were presented in a series of Figures (4, 5, and 6) that included the following subfigures:

1. Upper Subfigures (chemiluminescence Emission Signals): These plots show the chemiluminescence emission patterns of the flame under different conditions. These subfigures display how the emission intensity varies with time as the fuel excitation frequency and applied voltage to the load-speaker at zero voltage to the voltage just before the flame blowout. These plots help visualize how the

flame's chemiluminescence emission behaves under both stable and unstable conditions.

2. Middle Subfigure (Auto-Correlation): This plot illustrates the auto-correlation of the chemiluminescence signals. By showing the strength of the correlation at various time lags, these plots provide insight into the regularity or randomness of the flame's behavior. A steep drop to zero in the correlation function would indicate flame instability, while a consistent, high correlation suggests a stable flame.
3. Lower subfigures (Spectral analysis): This plot displays the frequency components of the chemiluminescence emission signal. Spectral analysis helps to identify whether certain frequencies dominate the signal, which can provide further clues about the flame's stability or instability.

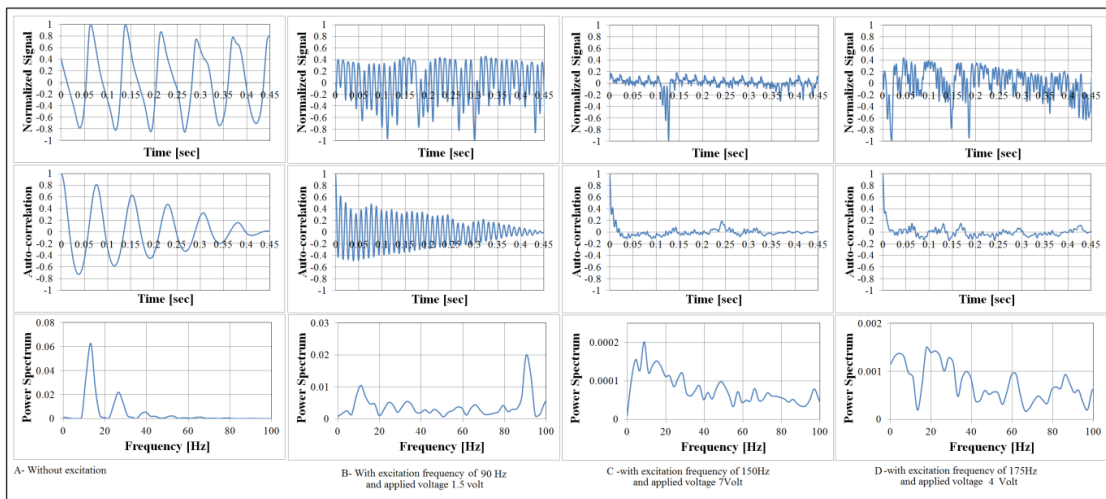


Figure (4): Free jet diffusion flame analysis with and without fuel excitation, diameter of 0.5 mm.

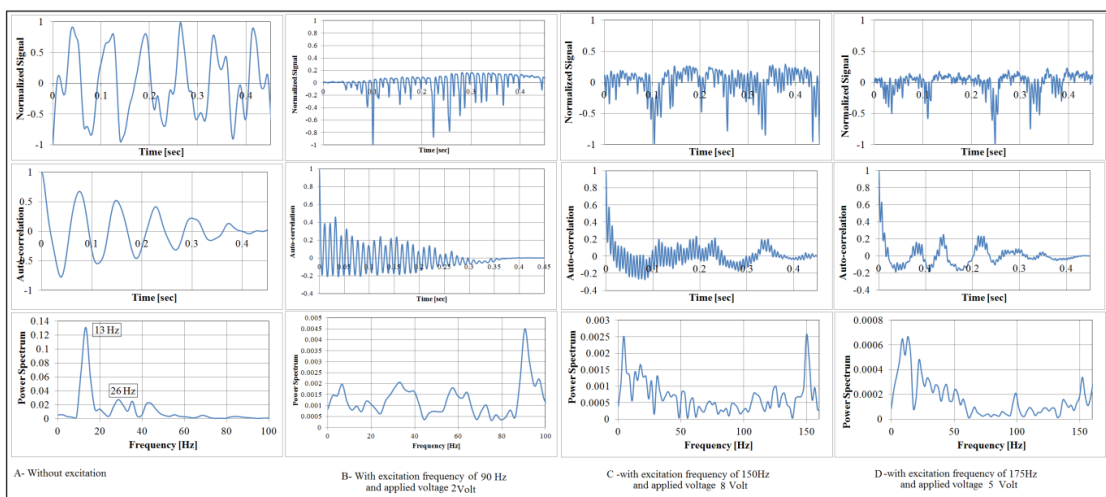


Figure (5): Free jet diffusion flame analysis with and without fuel excitation, diameter of 0.75 mm.

The results showed that at the burner diameter of 0.5 mm, as shown in Figure (4) when the fuel was excited at a frequency of 90 Hz, the flame was unstable. The auto-

correlation curve (which is a way of comparing a signal with itself to detect repeating patterns) without excitation shows a strong, stable peak around 13 Hz. This 13 Hz peak corresponds to the natural frequency of the laminar flame, this frequency at which the flame naturally oscillates when it's left undisturbed. It's like the flame has a "preferred" oscillation mode.

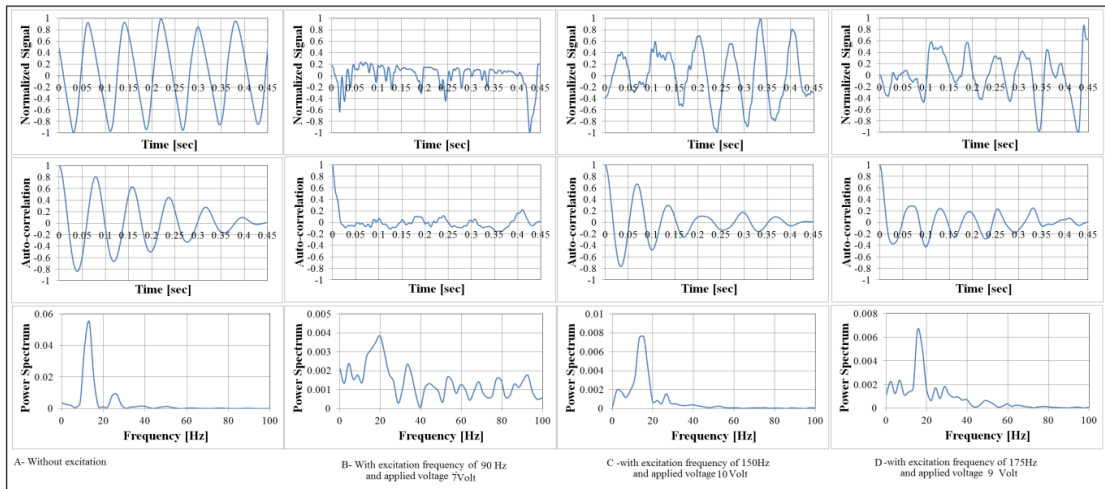


Figure (6): Free jet diffusion flame analysis with and without fuel excitation, diameter of 1.0 mm.

As the voltage supplied to the loudspeaker increases, the intensity of the acoustic signal rises accordingly. This results in stronger disturbances to the flame, causing its natural oscillations to be increasingly influenced and potentially altered by the external acoustic vibrations. When the applied voltage reaches 1.5 volts, the flame extinguishes, showing that the sound energy has become too much for the flame to handle. The auto-correlation becomes irregular, indicating the flame is no longer in a stable, predictable state. This irregularity is also shown in the spectral analysis of the flame's chemiluminescence emission, where multiple peaks emerge, signaling that the flame is behaving randomly and chaotically.

In Figure (4) at a frequency of 150 Hz, the flame is more resilient compared to 90 Hz. It takes a higher voltage (7 volts) to extinguish the flame, indicating that the flame is less sensitive to disturbances at this frequency. Essentially, the flame is "better equipped" to handle the sound excitation at this frequency without being destabilized as easily as at 90 Hz.

At a frequency of 175 Hz: This frequency also shows improvement in flame stability compared to 90 Hz, as the flame was extinguished at 4 volts. It still does not have as much stability as at 150 Hz, but the flame is more stable than it was at 90 Hz. This could be due to the flame being able to adapt to the higher frequency vibrations, which may cause fewer large-scale disturbances in the flame's structure.

Burner Diameter of 0.5 mm: A small burner diameter of 0.5 mm means the flame is more easily affected by external factors like sound excitation. At excitation frequencies, especially 90 Hz, the flame extinguishes with just 1.5 volts applied to the loudspeaker. This suggests that the smaller the burner, the more susceptible the flame is to fluctuations, and it struggles to maintain its stability under these conditions.

Burner diameter of 0.75 mm improves flame stability as shown in Figure (5). A larger diameter means that the flame is more stable, and this can buffer it from

disturbances. As a result, the flame can withstand a higher applied voltage before going unstable: it extinguishes at 2 volts at 90 Hz, 8 volts at 150 Hz, and 5 volts at 175 Hz. This suggests that the increased burner diameter helps to stabilize the flame.

With a further increase to 1.0 mm in the burner diameter as shown in Figure (6), the flame shows the greatest stability. At 90 Hz, it requires 7 volts to extinguish, at 150 Hz 10 volts, and at 175 Hz 9 volts. This significant increase in extinction voltage shows that the larger burner is even better at resisting the influence of external sound disturbances.

For comparison, the flame with a 0.5 mm burner extinguishes at much lower voltages: 1.5 volts at 90 Hz, 7 volts at 150 Hz, and 4 volts at 175 Hz. This shows the stark contrast in flame stability with a smaller burner diameter. Essentially, the larger burner diameter provides a more stable flame, and it requires more energy (sound-induced disturbance) to destabilize it.

In summary, the experiment demonstrates that flame instability is heavily influenced by both the excitation frequency and the burner diameter. A larger burner diameter offers greater stability, and at certain excitation frequencies reduces flame instability. By controlling these factors, it is possible to design more efficient and stable combustion systems that are less prone to disturbances.

The interaction between the acoustic waves and the combustion process can cause the flame to oscillate with increasing amplitude, and at a certain threshold, this instability may lead to the flame blowout. The results of the experiments and the spectral analysis of flame oscillations with varying burner diameters showed that the oscillation pattern remained constant; that is, the flame exhibited instability at the same frequencies for all diameters, and likewise showed stability at the same frequencies. However, a clear difference was observed in the level of flame stability while keeping the volumetric fuel flow rate constant, as the intensity of the instability increased with smaller burner diameters. This behavior is attributed to the higher Reynolds number associated with smaller diameters. In general, as the burner diameter increased, the flame exhibited greater stability. This suggests that larger diameters allow for better control over the combustion process, perhaps by providing a more uniform distribution of fuel and air, reducing localized hotspots that can lead to instability.

CONCLUSION

This study demonstrated that both burner diameter and acoustic excitation frequency are fundamental in governing flame stability. investigated the impact of burner diameter and acoustic excitation frequency on the instability of a free jet diffusion flame, with a particular focus on understanding thermo-acoustic instability a critical issue in continuous combustion systems such as gas turbines and jet engines. The experimental results clearly demonstrate that burner diameter significantly influences flame instability. Larger burner diameters were associated with more stable flame behavior, while smaller diameters exhibited increased susceptibility to thermo-acoustic instabilities. These findings are especially relevant to the design of combustion chambers in high-performance engines, where stable and efficient combustion is vital for performance and component longevity. Acoustic excitation frequency which was also found to play a crucial role in flame instability. The highest level of instability was observed at a frequency of 90 Hz across all nozzle sizes, At 90 Hz the flame exhibited the highest level of instability, characterized by erratic chemiluminescence signals and weak auto-correlation, indicating a chaotic combustion regime. At 175 Hz, the flame

showed moderate instability less chaotic than at 90 Hz but not as stable as at higher frequencies. The most stable flame behavior was observed at 150 Hz, where chemiluminescence emissions were highly correlated and regular, indicating a predictable and stable combustion process, the researchers were able to draw valuable conclusions about the relationship between nozzle size, fuel excitation frequency, and flame instability.

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DECLARATION OF CONFLICTING INTERESTS

The authors declare no conflict of interest.

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DECLARATION OF GENERATIVE AI AND AI-ASSISTED TECHNOLOGIES IN THE WRITING PROCESS

During the preparation of this work, the authors utilized AI-assisted tools to enhance language quality and grammatical accuracy, as well as to aid in translating certain paragraphs of the abstract into Arabic. Following the use of these tools (Chat GPT Premium), the authors meticulously reviewed and edited the content as necessary, and they assume full responsibility for the final version of the manuscript. The authors affirm that all experimental work was carried out exclusively in the Mechanical Engineering laboratories, and that the analysis and interpretation of results were conducted independently without dependence on AI-generated content.

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